MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 104 No. 4

MEETING OF 1944 JUNE 9

Professor E. A. Milne, President, in the Chair

The election by the Council of the following Fellow was duly confirmed:—

Elvet Lewis, R.A.F. Station, Boscombe Down, Amesbury, Wiltshire.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Harold Whitaker Browne, 24 Tamarisk Avenue, Reading, Berkshire (proposed by E. A. Beet);

Sydney Gerald Carpenter, 19 Farm Avenue, Streatham, London, S.W. 16;

Charles Donovon Grimwade, Capella, Parkway, Trentham, Staffordshire (proposed by P. F. Everitt);

John McLean Hildt, jun., U.S.N. 803, London Fleet, P.O. (proposed by G. C. L. Saul);

Renouf William Arnold Johns, The Mascot, Westward Ho!, Devon (proposed by G. W. Wakeford);

Philip Stevenson Laurie, 115 High Street, Chislehurst, Kent (proposed by H. Spencer Jones); and

John Gwynne Lloyd, B.Sc., 43 Upper Cimla Road, Neath, Glamorganshire (proposed by A. Buxton).

Fifty presents were announced as having been received since the last ordinary meeting.

THE RECESSION-CONSTANT OF THE GALAXIES

Sir A. S. Eddington, O.M., F.R.S.

(Received 1944 May 19)

Summary

Theoretical arguments suggested a relation between the size of the universe and the constants of atomic physics as early as 1919. The suggestion became more definite in 1930, when it was found that the ratio $2{\cdot}3 \cdot 10^{39}$ of the electrical to the gravitational force between a proton and electron was nearly the square root ($5{\cdot}3 \cdot 10^{39}$) of the number of elementary particles in the universe found from the observed recession of the nebulæ. There are now several independent methods of calculating the nebular recession-constant from laboratory data, and these agree on the definitive value 572 km. per sec. per mp. This is considered in conjunction with astronomical data as to the system of the galaxies. It appears that the "expansion" (ratio of the present radius to the Einstein radius) must be between 3 and 8. The former vague description of past time as logarithmically infinite is superseded by a definite upper limit of 90,000 million years.

The existence of a theoretical relation connecting the size of the universe with the fundamental constants of atomic physics was first suggested by H. Weyl about 1919.* There are two lengths associated with an electron in classical theory: its "electrical radius" $a_1 = e^2/m_e c^2 = 2{\cdot}8 \cdot 10^{-13}$ cm., and its "gravitational radius" $a_2 = \kappa m_e/c^2 = 6{\cdot}7 \cdot 10^{-56}$ cm., κ being the constant of gravitation. Guided by his unified theory of electricity and gravitation, Weyl thought it probable that, if R is the radius of space, R/a_1 has the same order of magnitude as a_1/a_2. In 1922 I suggested a slightly different form of connection.† Considering the ratio $a_1/a_2 = 4 \cdot 10^{42}$, it seemed difficult to account for this large number in the scheme of things unless it was somehow connected with the number N of elementary particles in the universe; and I thought it probable that a_1/a_2 was of order of magnitude $\sqrt{N}$. Actually the difference between the two suggestions was mainly in form, because it easily follows from the formula $\kappa M/c^2 = \frac{1}{2}\pi R_0$, connecting the mass M and radius R_0 of an Einstein universe, that R_0/a_2 has order of magnitude N; so that, if Weyl's suggestion is right, R_0/a_1 and a_1/a_2 each have order of magnitude $\sqrt{N}$.

"Order of magnitude" had to be interpreted elastically, because there was no reason for preferring the constants a_1, a_2 of the electron to the corresponding constants a_1', a_2' of the proton. The proton constants gave a size of the universe which was obviously much too small; the electron constants gave a size now known to be much too large. Presumably the suggestion could be given a sharper form by making a reasonable compromise. The ratio a_1/a_2 is $e^2/\kappa m_e{}^2$—that is to say, it is the ratio of the electrical force e^2/r^2 to the gravitational force $\kappa m_e{}^2/r^2$ between two electrons; the rival ratio $a_1'/a_2' = e^2/\kappa m_p{}^2$ is the corresponding ratio for two protons. We avoid bias by taking as the most fundamental constant of this kind the ratio of the electrical to the gravitational force between a proton and electron

$$F = e^2/\kappa m_p m_e = 2{\cdot}3 \cdot 10^{39}. \qquad (1)$$

I have called this the *force ratio.*

* I am unable to find the original reference. The suggestion is mentioned in *Space, Time and Gravitation*, p. 178, 1920.

† *Mathematical Theory of Relativity*, p. 167, 1922.

By 1930 astronomical measurements of nebular recession, combined with the Friedmann-Lemaître theory of the expanding universe, had given a fairly trustworthy determination of the mass of the universe, and hence of the number of protons and electrons composing it. The result was $N = 2{\cdot}8 \cdot 10^{79}$ *, so that

$$\sqrt{N} = 5{\cdot}3 \cdot 10^{39}. \qquad (2)$$

It is hard to understand the point of view of those who saw no significance in this agreement of the order of magnitude of F and $\sqrt{N}$. It seemed to me to leave no serious doubt that the observed nebular recession was the expected relativistic effect. What is more important, astronomical observation had, by establishing this agreement, given a clue which must certainly lead to a great unification of physical theory.

Following up this clue, my first attempt to find the precise connection between the cosmological constants and the constants of atomic physics was published in 1931.† The astronomical bearing of the result was discussed in *M.N.*, **92**, 3, 1931; and the main object of the present paper is to bring this discussion up to date. The recession-constant calculated in 1931 was 528 km. per sec. per megaparsec; subsequent corrections now make the value 572·4. Meanwhile advances have been made in our knowledge of the distribution and masses of the galaxies, which make it possible to give a provisional estimate of the present state of expansion of the system.

No advance was made on the 1931 theory until 1935.‡ It may be of interest to mention that the step then taken was also provoked by astrophysics; the clearing up of a serious discrepancy in the relativistic theory of white-dwarf matter led round to the cosmological problem, and provided a new way of attacking it. This led to the treatment adopted in *Protons and Electrons* (1935). In 1940, a discrepancy with the observational results in proton-scattering experiments, pointed out by H. M. Thaxton, led to the discovery of an error of a factor $(\frac{3}{4})^{\frac{1}{2}}$ in the 1935 formulæ.§ With this emendation, the result has been confirmed by a variety of subsequent investigations. The up-to-date theory is given fully in two recent papers.||

The well-known equation $\kappa M/c^2 = \frac{1}{2}\pi R_0$ for an Einstein universe leads at once to

$$R_0/N = \kappa(m_p + m_e)/\pi c^2 = 3{\cdot}95 \cdot 10^{-53} \text{ cm}. \qquad (3)$$

Our aim is to find a formula for $R_0/\sqrt{N}$ in terms of laboratory constants; we can then eliminate N, obtain R_0 separately, and hence calculate the recession-constant V_0 which by Lemaître's theory is $c/R_0\sqrt{3}$. My earliest formula (1931) was $e^2/m_e c^2 = R_0/\sqrt{\frac{1}{2}N}$, or in our previous notation $R_0/a_1 = \sqrt{\frac{1}{2}N}$. (The apparent bias towards the electron is illusory, because the m_e in the formula is more strictly the reduced mass $m_e m_p/(m_e + m_p)$ in the relative orbit of a proton and electron; so that the exact expression is symmetrical in m_p and m_e.) Later investigations have replaced $\frac{1}{2}N$ by $\frac{9}{20}N$, and introduced some more trifling modifications.

There are now four independent ways of calculating the recession-constant from laboratory data. By "independent" I mean that, apart from the present theory, we have no reason to expect agreement of the results. The methods are—

(1) From the Rydberg constant. (Paper I, chiefly §§ 23, 26.)
(2) From the range-constant k in the formula Ae^{-r^2/k^2} for the non-Coulombian energy of two protons. (Paper I, § 4.)
(3) From the energy-constant A in the same formula. (Paper I, § 28.)

* *M.N.*, **90**, 678, 1930; *Proc. Camb. Phil. Soc.*, **27**, 19, 1931.
† *Proc. Roy. Soc.*, A, **133**, 605, 1931.
‡ *Proc. Roy. Soc.*, A, **152**, 253, 1935.
§ *Physica*, **7**, 122, 1940.
|| *Communications of the Dublin Institute of Advanced Studies*, Ser. A, No. 2, 1944 (referred to as Paper I); *Proc. Camb. Phil. Soc.*, **40**, 37, 1944 (Paper II).

(4) From an entirely theoretical calculation of N without experimental data. The derived value of the recession-constant involves experimental data introduced by formula (3). (Paper II.)

The theory of method (2) is much simpler than the others; but the experimental value of k is as yet uncertain to about 2 per cent. The experiments give poor determinations of k and A, but an excellent determination of Ak^2; so that at present it is best to use a combination of (2) and (3). Method (4) is the most direct—if the reader is not frightened of epistemology. Method (1) is the amended 1931 method.

Thus, disregarding one method which is of inferior accuracy, three independent determinations of the recession-constant from laboratory data agree on the value 572. These form part of a comprehensive theory to which a great many other experimental tests (not directly concerned with cosmological constants) have been applied.* It therefore seems reasonable to accept the value 572 as definitive.

In one respect these investigations simplify Lemaître's theory. Setting $\lambda_0=(\frac{1}{2}\pi c^2/\kappa M)^2$, where M is the mass of the universe, it has been usual to distinguish three possible cases according as the cosmical constant λ is greater than, equal to, or less than λ_0. If $\lambda=\lambda_0$ an equilibrium state of the universe is possible, though it is unstable; otherwise equilibrium is impossible. I think that all early writers on the expanding universe (including myself) naïvely regarded the constants of nature as existing before the universe was created and the amount of its contents decided, and took it for granted that λ and λ_0 were independent constants. The present approach to the problem makes it clear that λ_0 *is* the cosmical constant. Thus the alternatives, which contemplate a universe without any possible static configuration †, do not arise.

If R is the actual radius of the expanding universe at any time, we call the ratio

$$q=R/R_0 \tag{4}$$

the *expansion*. The rate of recession V at that time is less than the recession-constant V_0 by an amount depending on q, the formula being

$$\frac{V}{V_0}=\left(1-\frac{3}{q^2}+\frac{2}{q^3}\right)^{\frac{1}{2}}, \tag{5}$$

and the density ρ is related to the density ρ_e of the equilibrium state by

$$\rho=\rho_e/q^3. \tag{6}$$

As these formulæ refer to a spherical space, it is implied that the galaxies can be treated as having large-scale uniformity of distribution. The region now surveyed covers a considerable part of the whole space and, so far as it goes, supports the assumption. No hypothesis is made as to the initial state of the universe; but, since the possibilities are now restricted to $\lambda=\lambda_0$, it follows that if the time is continued far enough back the Einstein equilibrium state is approached.

For the measurement of large-scale density it is convenient to introduce a unit of 10^{10} times the Sun's mass per cubic megaparsec, which I will call X mp. For the equilibrium state, the definitive value $V_0=572{\cdot}4$ gives

$$M=1{\cdot}977\,.\,10^{55}\text{ gm.}=10^{22}\,\odot.$$
$$R_0=9{\cdot}335\,.\,10^{26}\text{ cm.}=302{\cdot}4\text{ megaparsecs}=9{\cdot}895\,.\,10^{8}\text{ light-years.}$$
$$\rho_e=1{\cdot}231\,.\,10^{-27}\text{ gm. cm.}^{-3}=1824\text{ X mp.}$$

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* In all there are fourteen tests.

† If equilibrium is a violation of the laws of nature, ought not every reference to equilibrium to be struck out of the definitions of physical quantities? The elimination of the universes with $\lambda\neq\lambda_0$ relieves us from an awkward logical dilemma.

By (5) and (6) we obtain the following results for different assumed values of q:—

q	V	ρ (gm. cm.$^{-3}$)	ρ (X mp.)
∞	572·4	0	0
10	564·3	$1{\cdot}23.10^{-30}$	1·82
8	559·9	$2{\cdot}40.10^{-30}$	3·56
6	550·8	$5{\cdot}70.10^{-30}$	8·45
5	541·8	$9{\cdot}85.10^{-30}$	14·6
4	525·8	$1{\cdot}92.10^{-29}$	28·5
3	492·6	$4{\cdot}56.10^{-29}$	67·6
2·5	460·7	$7{\cdot}88.10^{-29}$	117
2	404·7	$1{\cdot}54.10^{-28}$	228

This table, used in conjunction with astronomical data, enables us to estimate the present value of q. Hubble and Humason's value of V is 560. A considerable margin for error must be allowed; but it is perhaps not unduly optimistic to regard this as excluding a value less than 490. That gives $q > 3$. According to Shapley * there are 5 or 6 galaxies per cubic megaparsec. There is general agreement that the mass of our own galaxy is about $10^{11}\odot$ †; but the average galaxy is somewhat smaller. It is not very clear what the difference is estimated to be, but I think that a mass $10^{10}\odot$ for an average galaxy would be regarded as a lower limit. That gives a density not less than 5 X mp.; so that (by the table) $q < 8$. Hubble ‡ gives the density as 10 galaxies per cubic megaparsec, and adds, "these nebulæ average about 100 million times brighter than the Sun and several thousand million times more massive". This would agree with $q = 8$. In view of the upward tendency of calculations of masses, and the possibility of inter-galactic matter (referred to below), I think the best present estimate of q would be about 5.

We conclude that the present radius of the universe is between 1000 and 2500 megaparsecs, and most probably about 1500 megaparsecs.

On any evolutionary hypothesis it is unlikely that the space between the galaxies is entirely empty. By analogy with the cosmic cloud in our own galaxy we might expect the amount of matter left uncondensed to be roughly the same as the amount condensed into galaxies. It cannot be very much greater. Assuming it to be observationally established that $V > 490$, we have $\rho < 67$ X mp. Allowing 5 galaxies per cubic megaparsec, the total mass is not greater than $1{\cdot}3.10^{11}\odot$ per galaxy.

Since $M = 10^{22}\odot$, this result shows that the total number of galaxies is not less than 7.10^{10}. A probable estimate is about 3.10^{11}.

Adopting $q = 5$ as the present value, we can determine the dates when the universe had, or will have, other values of q. Setting $x^2 = q/(q+2)$, the formula is §

$$\tfrac{1}{2}ct/R_0 = \sqrt{3}.\tanh^{-1} x - \coth^{-1}(x\sqrt{3}) + \text{constant}. \qquad (7)$$

The results are:

q	ct/R_0	t (10^9 years)
20	2·447	2·42
10	1·237	1·22
5	0·000	0·00
2	− 1·861	− 1·85
1·2	− 3·679	− 3·65
1·0727	− 4·730	− 4·68 (*a*)
1·0030	− 7·941	− 7·86 (*b*)
$1+10^{-39}$	−91·94	−91·0 (*c*)

* *Galaxies* (Blackiston Co., 1943), p. 189.

† A recent determination by Oort from the distribution and velocities of RR Lyræ variables gives $10^{11}\odot$ (*Monthly Astronomical News Letter*, No. 18).

‡ *Smithsonian Report*, 1942, p. 121.

§ Lemaître, *M.N.*, 91, 488, 1931.

Notes on the last three Entries

(*a*) This is the date when the universe burst.* Previously it was ideally possible for a ruler, issuing orders by radio, to influence future events in *every* part of the universe. Orders issued at the time t_a do not reach an antipodal galaxy until $t = \infty$. After t_a parts of the universe are entirely beyond the spheres of causative influence of other parts.

(*b*) On this date it ceased to be possible for light to travel right round the universe.

(*c*) This is the extreme limit of past time. Owing to statistical fluctuations, the scale of length cannot be defined more closely than 1 part in 10^{39} (Paper I, § 2). On the date t_c the universe was actually in the Einstein state, since R was within the limits of statistical fluctuation of the Einstein radius R_0. (The nominal difference $R - R_0$ was no more than 10^{-13} cm.)

By (7) the time taken by the universe to reach its present expansion, if it started from a *geometrically* uniform equilibrium state, is logarithmically infinite. The calculation (*c*) replaces this by the more useful conclusion that the time taken, if it started from a *statistically* uniform equilibrium state, is about 90,000 million years. The conditions corresponding to the extreme limit are, of course, a fantastic extrapolation; but we do not reduce the time limit very much by cutting out the more extravagant part of the solution, and the system can be traced back for, say, 30,000 million years without straining credulity. If at that time the condensations, ultimately to develop into separate galaxies, were already formed, the time allowance is fairly satisfactory; but there is certainly insufficient time if we try to include in this period (or in the extreme period of 9.10^{10} years) evolution of the condensations by gravitational instability of a uniform primordial distribution. To this it is perhaps a valid answer that we are still without any explanation of the origin of the rapid rotation of the galaxies; and it is premature to consider the time demands of an evolutionary theory which is bound to undergo modification on that account.

Observatory, Cambridge:
1944 *May* 17.

* *The Expanding Universe*, p. 74.

SOME RESULTS OF A SPECTROPHOTOMETRIC STUDY OF THE WOLF-RAYET BINARY HD 193576

C. S. Beals

(Received 1944 June 10)

Summary

Values of equivalent width are given for all readily observable absorption line and emission bands of this star in the region λ 3900–6678. Comparison of line and band intensities with those of typical stars of the same spectral classes have led to apparent visual magnitudes of the two components of the binary of 8·30 for the O-type star and 10·05 for the W companion. The corresponding absolute magnitudes calculated on the basis of interstellar line intensities are −3·07 and −1·32.

The asymmetrical character of the emission bands at primary and secondary minimum is shown to be shared by λ 4686 and members of the Balmer and Pickering series. The asymmetry is attributed to tidal action within an extensive envelope, and this interpretation is strengthened by the behaviour of the displaced absorption lines due to *He*I and *He*II. Observations of these lines indicate *He*I and *He*II envelopes of radii larger and smaller respectively than the distance between the two stars, suggesting that Wolf-Rayet emission arises in an envelope large relative to the central star.

It is pointed out that the asymmetry of the emission bands introduces difficulties into the application to this star of the theory of the "transit-time" effect developed by Wilson, which predicts an observable difference between the times of eclipse obtained by spectrographic and photometric observations.

In 1940 O. C. Wilson published the orbital elements of the spectroscopic binary HD 193576 *, α (1900), $20^h\ 15^m.8$, δ (1900), $+38°\ 25'$, $m_v = 8·1$, and showed that one of the components was a Wolf-Rayet star of early spectral type, while the other was an absorption line star which he classified as B1. Later Gaposchkin †, on the basis of Harvard photographic observations, derived a light curve, showed that the star was an eclipsing variable and computed the absolute dimensions of the system. More recently Wilson ‡ has published revised values of the absolute dimensions of the system basing his results on an unpublished light curve by Kron § and on variations with phase of the absorption line *Hγ*.

Wilson also discussed in some detail the question of the origin of Wolf-Rayet emission. He pointed out that in an envelope which was large relative to the central star the length of time required for an atom to reach the region of emission would result in an appreciable difference between the radial velocity of the star and that of the envelope as indicated by the positions of the band centres. Such a difference would show itself in a discrepancy (designated by Wilson as the transit-time effect) between the spectroscopically predicted eclipse and the true eclipse time as determined by photometric observations. The absence of such a discrepancy in the observations of HD 193576, combined with the lack of such occultation effects as would be expected in a small envelope of the same

* *Ap. J.*, 91, 379, 1940.
† *Ap. J.*, 93, 202, 1941.
‡ *Ap. J.*, 95, 402, 1942.
§ Later published *Ap. J.*, 97, 311, 1943.

order of size as the central star, led Wilson to question the validity of the commonly accepted ejection hypothesis as an explanation of the Wolf-Rayet emission features.

Prior to the publication of Wilson's results a number of spectrograms of HD 193576 had been obtained by the Director and by the writer at this observatory. A study of these spectrograms suggested that a somewhat different approach to the problem might lead to an independent determination of the relative brightness and diameters of the two components of the system. Also the association of a W star with an absorption line object in a binary system appeared to present interesting possibilities for the study of tidal effects and absorption effects which might have an important bearing on the theory of Wolf-Rayet emission. A series of spectrographic observations was accordingly undertaken in the 1942 summer season, and this paper deals with spectrophotometric results derived from these observations. The data on which the results are based consist of 61 spectra, 39 of which cover the region λ 3900–5000, while 22 are in the region λ 5300–6700. Spectra in the short wave-length region were secured with one prism and dispersions at $H\gamma$ of 51 and 30 A. per mm. In the visible region both single- and three-prism arrangements have been used with dispersions ranging from 46 to 230 A. per mm. at $H\alpha$.

The Intensity Measures

The spectrophotometric measures have included all observable emission bands and absorption lines in the region λ 3900–6678. In most instances absorption lines are blended with emission bands, but fortunately the great width of the emission bands

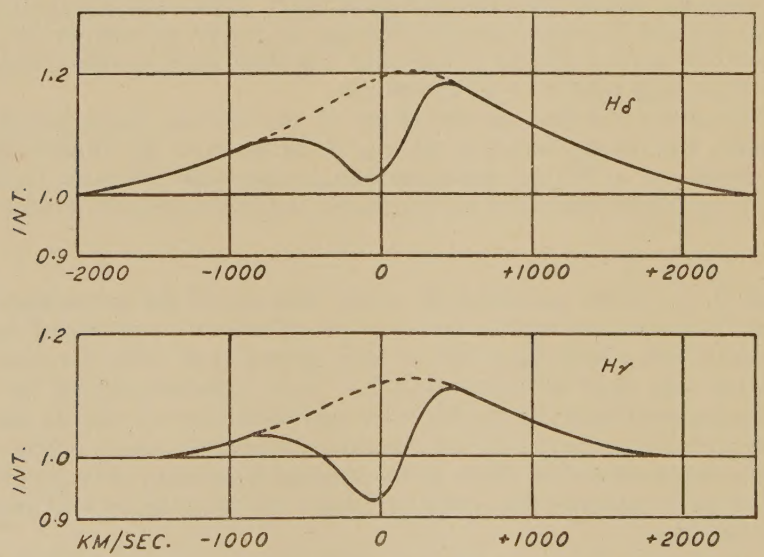

FIG. 1.—*Profiles of $H\gamma$ and $H\delta$, at phase 0·301. The greater strength of emission $H\delta$ is believed to be due to a blend with* NIII.

usually makes it possible to derive separate profiles for each. The profiles of $H\gamma$ and $H\delta$ in Fig. 1 illustrate the blending effect and the difficulties which are encountered in securing accurate measures of intensity. The full line represents the observed profile, while the position of the dotted line, which must be sketched in, depends to a considerable extent upon the personal judgment of the observer.

The spectrophotometric work (which has involved the determination of over 500 separate line profiles) has been greatly facilitated by a newly built instrument for the mechanical reduction of microphotometer tracings. This instrument is described in a

separate publication *, and for present purposes it suffices to say that the labour of deriving equivalent widths from microphotometer tracings is reduced by a factor of the order of 10 without loss of accuracy.

Table I gives the wave-lengths, identifications and mean equivalent widths of emission bands, while Table II contains similar information for the absorption lines outside of eclipse. Intensities are given for 16 emission bands. In all cases a correction, based on the light curve, has been applied to make the intensities comparable at different phases. The average values given in Table I have been computed on the assumption that the emission is constant and not appreciably affected by the eclipses. A number

TABLE I

Emission Band Intensities, HD 193576

λ	Atom	No. Plates	Equivalent Width km./sec.
6678	*He*I	3	181
6563	*Hα* + *He*II	4	1243
5875	*He*I	4	159
5806	*He*II	4	498
5411	*He*II	4	563
4940	*N*IV + *N*V	1	50
4861	*Hβ* + *He*II	33	361
4686	*He*II	25	3277
4603	*N*V	33	175
4340	*Hγ* + *He*II	33	203
4200	*He*II	33	153
4101	*Hδ* + *He*II + *N*III	33	468
4057	*N*IV	33	368
4025	*He*II + *He*I	29	148
3970	*Hε* + *He*II	5	124
3925	*He*II	1	36

of other bands are known to be present but for them blending makes it impossible to derive reliable intensities. Twelve absorption lines are given in Table II, but these include four lines of interstellar origin. The most reliable absorption-line intensities are those of *Hγ* and *Hδ*. λ 4200 of *He*II has been measured on 32 of the 39 plates, but its relatively small intensity gives it somewhat less weight. The *He*II absorption line

TABLE II

Absorption Line Intensities, HD 193576
Outside Eclipse

λ	Atom	No. Plates	Equivalent Width km./sec.
5895	*Na*I	1	21 †
5889	*Na*I	1	24 †
4861	*Hβ* + *He*II	12	111
4471	*He*I	16	37 ‡
4471	*He*I	7	25
4430	Unidentified	35	178 †
4340	*Hγ* + *He*II	13	117
4200	*He*II	13	57
4101	*Hδ* + *He*II + *N*III	13	104
4026	*He*II + *He*I	10	52
3970	*Hε* + *He*II	6	136
3933	*Ca*II	21	28 †

* *J.R.A.S. Can.*, 38, 65, 1944. † Interstellar. ‡ Displaced line.

at λ 4541 is detectable on most plates, but the blended character of the emission in this spectral region makes it difficult to secure reliable intensities. The *He*I line at λ 4472 due to the absorption-line star has been measured on 7 plates, while a second absorption line, displaced 400 km./sec. to the violet but believed to be due to the same atom, has been measured on 16 plates. The absorption line at $H\beta$ has been measured on practically all plates, but the steepness of the slope of the sensitivity curve of the photographic emulsion in this region makes it difficult to draw in the profile accurately. In the violet region numerous measures are available of the *He*II absorption line at λ 4026, but possible blending with *He*I, λ 4025, makes the intensities unreliable. A few scattered measures are available for the line $H\epsilon$, but the weakness of most spectra in this region and blending with interstellar *H* interferes with the accuracy of the measures.

Classification of the Emission and Absorption Line Spectra

The Emission Line Star.—The general appearance of the emission spectrum is that of a Wolf-Rayet star of the nitrogen sequence with rather weak bands. While the contrast with the continuous spectrum is relatively low it is important to note that the Wolf-Rayet spectrum is very completely developed and none of the bands associated with a normal W spectrum appears to be absent. In a composite spectrum the usual line intensity ratios cannot be used directly, and it is necessary to use more general considerations in fixing the type of the star. The presence of strong bands due to *N*V at λ 4603–19 suggests that the type is earlier than WN6, but the fact that the diffuse band with centre at λ 4640 (usually identified with *N*III and associated with class WN6) is still present in considerable strength indicates that the spectrum is of somewhat lower excitation than WN5. The relatively slow decrement in the Balmer-Pickering series also indicates a type somewhat later than WN5, and it would appear that WN5·5 should closely represent the true spectral type. It is important to fix the spectral class as precisely as possible since one method of fixing the relative brightness of the two stars depends upon it.

The Absorption Line Star.—While the absorption-line star was originally classified as B1 by Wilson, the appearance in its spectrum of relatively strong lines of *He*II at λ 4541 and λ 4200 left little doubt that it is actually an O-type star.

Estimates of its classification depend upon the relative intensities of *He*I and *He*II in the spectrum. The lines ordinarily employed are λ 4471 of *He*I and λ 4541 of *He*II. In the present instance λ 4541 is obscured by the blending of emission bands in this region and intensities are consequently unreliable. It appears, however, that the line λ 4541 is probably stronger than λ 4471, and if the line λ 4200 of *He*II be substituted the ratio of equivalent widths is λ 4471/4200 = 0·5. While the probable error of the ratio is doubtless fairly high, it is reasonably certain that the intensity of λ 4200 is considerably greater than that of λ 4471. An examination of Williams' spectrophotometric data on the O and B stars as well as similar measures made at this observatory makes it clear that for a value of the above intensity ratio less than unity the star should be classified O6 or earlier. H. H. Plaskett's * original classification of the O-type stars based on eye estimates of line intensity give the ratio λ 4471/4541 = 0·8 for class O6, and since λ 4200 is ordinarily of smaller intensity than λ 4541 the star can scarcely be classified later than O6 and may even be somewhat earlier. For the purposes of the present study a value of O6 has been adopted, with an estimated probable error of not more than half a subdivision.

The Relative Brightness of the Two Components of the Binary.—There are several methods of using spectrophotometric measures of line intensity to derive the relative brightness of the two binary components. One such method, already employed by Wilson, is to make use of the relative intensities of absorption lines in and out of principal

* *P. Dom. Ap. O.*, 1, 325, 1922.

eclipse in connection with the equation,

$$q = \frac{1 - (W_p/W_0)}{A_p - 1}, \tag{1}$$

where q is the ratio of brightness of the two stars, W_p and W_0 are the equivalent widths in central eclipse and outside of eclipse, respectively, while A_p is the amplitude of principal minimum. Unfortunately the blending of emission bands and absorption lines leads to high values of the probable errors in individual measures, and the present series of observations has proved insufficient to determine, with the necessary precision, the relatively small variation of absorption-line intensity in and out of eclipse. In order to illustrate the difficulty, values of equivalent width for $H\delta$ have been plotted against phase in Fig. 2. While there is some indication in this figure of the expected variation

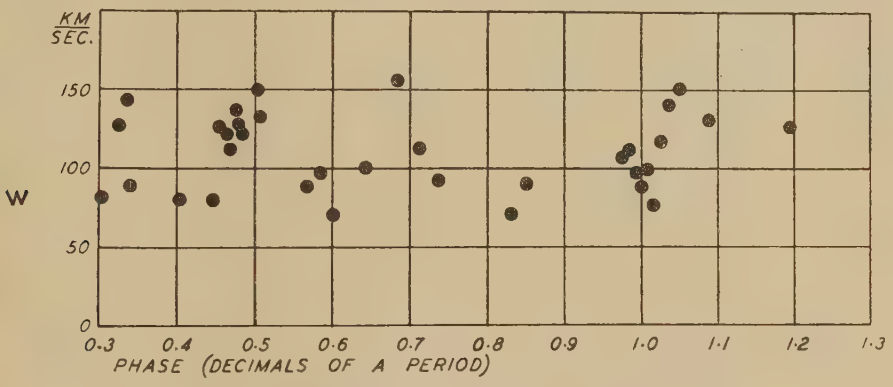

FIG. 2.—*Equivalent width of Hδ, plotted according to phase.*

of intensity in and out of eclipse, the variation is small compared to the probable error of the determination, and it appears that some other method must be employed if significant determinations of the relative brightness of the component stars of the binary are to be obtained from the data of Tables I and II. An alternative method depends upon the following relationships between mean equivalent width outside of eclipse and the relative brightness of the two stars making up the composite spectrum.

Consider first an absorption-line star and let the intensity of the continuous spectrum in the neighbourhood of an absorption line equal unity, and the intensity at a particular velocity displacement v from the centre of the line equal I_{ov}. Then if the continuous spectrum of a second star having a relative brightness q is superimposed upon the line, I_v, the intensity at the wave-length corresponding to velocity, v, will be represented by the following familiar equation:—

$$I_v = (I_{ov} + q)/(1 + q) \tag{2}$$

or

$$q = (I_v - I_{ov})/(1 - I_v). \tag{3}$$

Making use of line depths rather than intensities equation (3) becomes

$$q = \frac{D_{ov}}{D_v} - 1, \tag{4}$$

where D_o and D are the line depths in the undisturbed line and in the composite spectrum respectively. Integrating over the width of the line a similar expression is derived for equivalent widths, E,

$$q = \frac{E_o}{E} - 1. \tag{5}$$

If emission rather than absorption is involved the expression is the same, remembering that in each case q represents the intensity of the spectrum superimposed upon the line.

Making use of (5) and treating the absorption and emission separately it is possible by making logical assumptions as to the line intensities corresponding to the particular spectral types involved, to estimate the relative brightness of the two stars. It is fortunate that in this case the absorption and emission lines differ so widely in form and intensity that it is possible to make independent measures of intensity for each.

Values of q from Emission Bands.—The emission-line star has been classified W5·5 and the complete development of its emission-line spectrum suggests that the true intensities of its emission features should be close to those of the typical stars of the class. The two typical stars for subclasses W5 and W6, HD 187282 and HD 192163, are listed by Beals * who also gives equivalent widths for a number of the bands. These measures were made before the registering microphotometer became available at this observatory, and for the purposes of this paper a number of the older plates have been remeasured and new data have been added to make the list of intensities considerably more complete. The values of emission-line intensity for HD 187282 and HD 192163 are given in Table III. Column 1 contains the values for HD 187282, while the intensities for HD

TABLE III

Relative Brightness of Components from Emission Bands, HD 193576

λ	E.W. HD 187282 WN5	E.W. HD 192163 WN6	E.W. Mean	E.W. HD 193576	q	Wt.
6563	6150 (2)	7854 (2)	7002	1289	4·44	(4)
5875	127 (1)	1736 (2)	931	109	7·54	(1/2)
5411	1380 (2)	3801 (2)	2590	620	3·17	(3)
4859	1148 (3)	2973 (3)	2060	362	4·70	(3)
4686	10525 (1)	14000 (1)	12267	3277	2·75	(1)
4603	959 (3)	1168 (1)	1063	175	5·14	(4)
4340	942 (3)	2029 (3)	1486	203	6·32	(3)
4200	320 (1)	1253 (3)	786	153	4·14	(1)
4101	745 (1)	5856 (3)	3300	468	6·06	(1/2)
4057	375 (1)	3887 (3)	2131	369	4·77	(1/2)

Weighted mean value of $q = 4·73$.
$\Delta m = 1·7$ magnitudes.

192163 are given in column 2. Column 3 gives the mean for the two stars, and this value is used for the calculation of q from equation 5. Band intensities for HD 193576 are given in column 4. Actual values of q for separate bands are given in column 5. Weights are assigned partly on the basis of a judgment of the reliability of the intensity measures and partly on the basis of the rate of variation of band intensity with spectral class. The weighted mean value of q from Table III is 4·91, indicating a luminosity of the absorption-line star 1·7 magnitudes brighter than that of the W-type component of the binary. Fig. 3 illustrates the approximate intensity of emission $H\gamma$ in HD 193576 and in the typical star.

Values of q from Absorption Lines.—The class of the absorption-line star has been set at O6 and the values of the line intensities for a typical star of this class have been determined from measures of line intensity for 12 O-type stars by Williams †, combined with an additional 10 different O-type spectra measured at this observatory.‡ While a number of lines due to hydrogen, helium I and helium II have been utilized in the computations, greatest weight is attached to the two hydrogen lines $H\gamma$ and $H\delta$, which were measured under much more favourable conditions than the others.

* *P. Dom. Ap. O.*, **6**, 95, 1934.
† *Ap. J.*, **83**, 279, 1936.
‡ R. M. Petrie, *Proc. Roy. Soc. Can.*, May 1944.

The results are given in Table IV. Column 1 contains the wave-lengths, column 2 gives the line intensities for the typical star, while column 3 gives the values of line intensities for HD 193576. Values of q are found in column 4. Since the absorption line in each case is superimposed upon a broad band the value of q in equation (5) refers to the intensity in the continuous spectrum of the W star plus the intensity of the band. In order to correct to the continuous spectrum it is necessary to divide by the combined intensity. Values of continuous spectrum + band are given in column 5. They represent

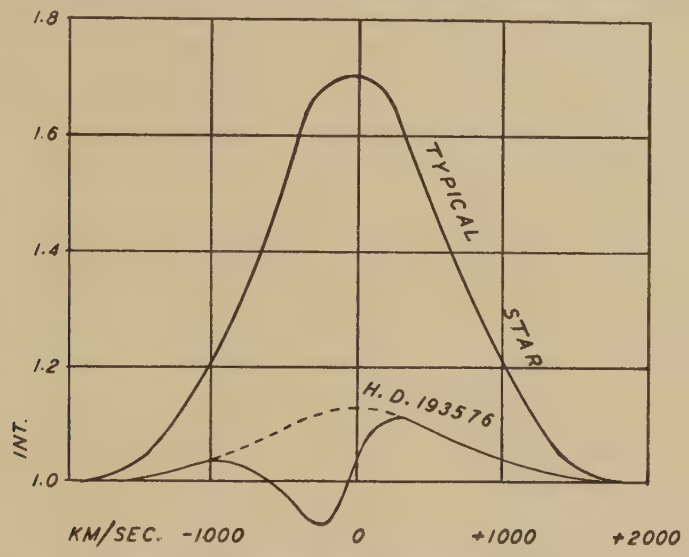

Fig. 3.—*Profile of Hγ, compared with computed profile of typical star.*

estimates based on band profiles computed for the typical star, and they take into account the average displacement of the absorption line from the band centre. Such a procedure is necessarily approximate, but the errors which it may involve are unlikely to affect the order of magnitude of the result.

Table IV

Relative Brightness of Components from Absorption Lines, HD 193576

λ	E.W. km./sec. Typical Star	E.W. km./sec. HD 193576	q	Correction Factor	Final q	Wt.
4861	150	110·6	0·35	1·7	0·21	(1)
4471	32	24·7	0·30	1·2	0·25	(1/2)
4340	135	117·4	0·26	1·45	0·18	(4)
4200	55	57·2	−0·04	1·27	not used for mean	
4101	128	103·6	0·24	1·35	0·18	(4)

Weighted mean value of $q = 0{\cdot}187$.
$\Delta m = 1{\cdot}8$ magnitudes.

The final corrected values of q in column 6 lead to a value of 0·187, which should equal the reciprocal of q in Table III. This corresponds to a luminosity of the absorption-line star of 1·8 magnitudes brighter than that of the W-type component.

The values of relative luminosity from these two tables refer to the continuous spectra of the two stars and no corrections have been made for possible differences in spectrophotometric gradient. The agreement shown by the results for the emission and absorption lines suggests confidence in the mean value of 1·75 magnitudes for the difference in magnitude between the two stars. It may be noted that this value is in the opposite sense from that of Wilson, who found the W star 0·38 magnitude brighter than

its O-type companion. On the other hand it agrees well with Gaposchkin's * value of 1·89 derived from his photographic light curve, making the O star the brighter of the two.

Absolute Magnitude and Diameter.—In order to calculate the absolute magnitudes of HD 193576, estimates of the distances have been made on the basis of the intensities of the interstellar lines. H and K of calcium lead to a distance of 875 parsecs. The intensity of the broad interstellar line at λ 4430, on the other hand, leads to a distance of 1800 parsecs. The intensity of λ 4430 is probably better determined than for any single star so far observed. The points on the curve relating intensity of λ 4430 with distance, published by Beals and Blanchet †, show a very large scatter, but it is uncertain whether the scatter is due to errors of measurement of this very diffuse line or to actual physical conditions in interstellar space. The value adopted for the distance, 1180 parsecs, represents a mean in which λ 4430 has been given half-weight as compared with the sharp interstellar lines.

Making use of this value of the distance, with a correction of 0·85 magnitude per thousand parsecs for general absorption, absolute magnitudes have been computed for the two components of the system. The diameter of the O star has been derived on the assumption that its temperature is 40,000°, while the temperature assigned to the W star is 80,000°. The results are shown in Table V where the previous values due to Wilson and Gaposchkin are also included. The value for the diameter of the W star is necessarily uncertain because of ambiguity in the effective temperature. Gaposchkin's value of 13,000° was obtained from his solution of the light curve on the assumption of a disk of uniform brightness. It is difficult, however, to accept 13,000° as the true average surface temperature of the W star. It is possible that we are dealing with an extended photosphere and a heavy darkening to the limb. It is also possible that the outer parts of the photosphere are partially transparent to the light of the O-type star. The form of the photo-electric light curve gives some credence to such a view, and Kron has suggested that the difference in width of the two minima and the absence of a strong reflection effect may be due to the presence of an envelope capable of absorbing the continuous spectrum of the companion star.

TABLE V

Dimensions of Components of HD 193576

	Gaposchkin	Wilson	Kopal	Beals
W Star	5·8⊙	18·5⊙	5– 3⊙	1·3⊙
O Star	9·7⊙	11·1⊙	8–10⊙	4·2⊙

Gaposchkin's values of diameter were derived from assumed values of absolute magnitude and temperature. Wilson's and Kopal's diameters are from solutions of the photometric light curve.

It is clear from the data of Table V that there is a definite discrepancy between the diameter of the O star derived from solutions of the light curve and that calculated from the absolute magnitude and temperature. Since the value of 40,000° adopted as the temperature of an O6 star must be regarded as conservative, there appears to be no obvious explanation of the discrepancy. It is possible that it may be a consequence of excessive localized space absorption. Fortunately a twofold or threefold increase in the stellar diameters will not affect the general conclusions of this paper.

Kopal in a private communication has presented a solution of the photometric orbit in which the greater duration of primary eclipse (double that of the secondary) is explained on the assumption that the W star is surrounded by a semi-transparent envelope capable of absorbing light from the O-type star. He suggests that over 90 per cent. of the light of the continuous spectrum of the W star is emitted by a bright core of relatively small diameter, and that while the envelope may emit a faint continuous spectrum its main

* *Loc. cit.*

† *M.N.*, 98, 398, 1938.

influence on the light curve is that of absorption. The diameter calculated for the W star refers to the bright core and is based on an assumed temperature of 80,000°. This value is an estimate by Beals * based on a discussion of temperatures of W stars determined by Zanstra's method of ionization and combination. A recent paper by Aller †, using a different method, gives somewhat lower temperatures, but the writer prefers for the present to make use of the earlier value. The uncertain character of the effective temperatures for these stars should be emphasized, but within fairly wide limits the values adopted will not greatly alter the present discussion.

Unsymmetrical Profiles of Wolf-Rayet Bands.—Wilson's measures of λ 4686 of *He*II covered all phases and indicated among other things that the band is about 18 per cent. weaker in the vicinity of primary minimum than at the opposite phase. He was unable to detect any effect of eclipse in the band intensity when the O-type star passed in front

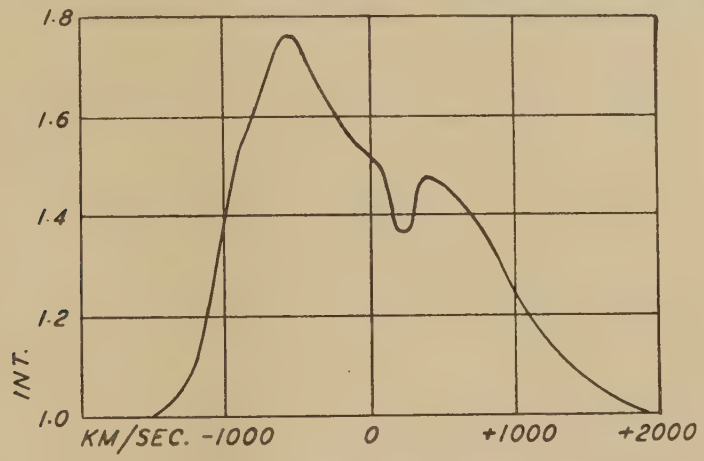

Fig. 4.—*Profile of Hγ, at phase 0·51 (near secondary minimum), showing peak displaced toward the violet.*

of its companion (secondary minimum). He also noted regular changes with phase of the form of the band. In the vicinity of primary minimum (phase 0·0) when the W star is nearest the observer the band is definitely unsymmetrical, with its peak shifted to the red. Near secondary minimum the peak is shifted to the violet, while at phases 0·25 and 0·75 the band has an approximately symmetrical form.

The primary aim of the present series of observations was to secure accurate measures of the intensities of the weaker bands and the absorption lines. As a consequence λ 4686 was over-exposed on a majority of the plates, and the results are accordingly inconclusive in the matter of determining the variation with phase of the intensity of this very strong band. Wilson's results, pointing to changes of the form of the band with phase, are, however, amply confirmed. An additional result of the present study has been to show that the behaviour of λ 4686 is in this respect closely correlated with that of the members of the Balmer and Pickering series. Fig. 4 shows the profile of *Hγ* from a plate taken near phase 0·5 showing the maximum of the band strongly shifted to the violet. A corresponding shift to the red is exhibited near primary minimum (phase 0·0). While later members of the series are weaker and in some cases blended with absorption lines it is reasonably certain that they all show similar variations in shape with phase.

While it is not the purpose of this paper to discuss this aspect of the subject in detail, it would appear that the most probable cause of the asymmetry in the band profiles is some form of tidal effect due to the presence of the massive O-type star as the other component of the binary. (Relative masses, according to Wilson, are W star 12·4⊙, O

* *P. Dom. Ap. O.*, 6, 9, 1934. † *Ap. J.*, 97, 135, 1943.

star 31·8 ⊙.) Such a tidal effect might be due in part to a stimulation of ejection from the hemisphere of the W star nearer its O-type companion and in part to an actual distortion of the trajectories of atoms ejected from the W star. The large mass of the companion is favourable to such a view, and certain absorption phenomena to be mentioned later also support the idea of an unsymmetrical envelope.

*Variation in Intensity of Displaced He*I *Absorption.*—An interesting feature of the spectrum of HD 193576 revealed by the microphotometer tracings is the presence of an absorption line about 400 km./sec. to the violet of the normal position of λ 4471 of helium. The helium line thus appears double with the component in the normal position, presumably due to the O-type star. It would appear logical to attribute the displaced component to absorption taking place in the expanding envelope since such a displaced line, due to *He*I, is a normal feature of many Wolf-Rayet stars. A remarkable feature of the displaced component is the fact that its intensity reaches a maximum value in the vicinity

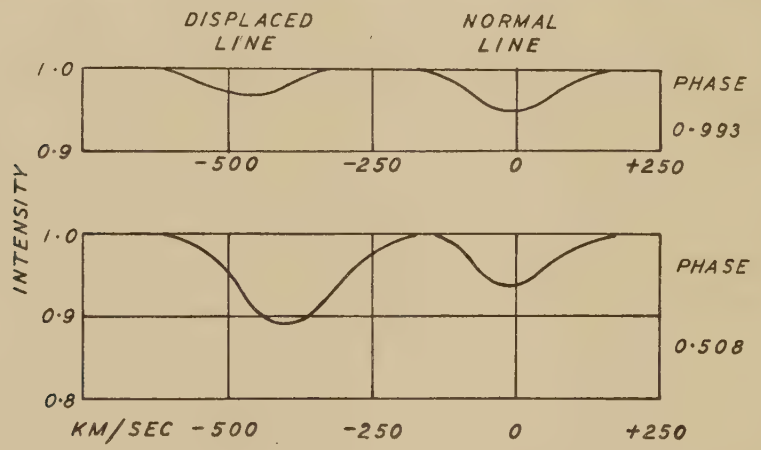

Fig. 5.—*Variation of intensity with phase of displaced line of λ4471 He*I (*attributed to the envelope*) *compared with the undisplaced line* (*attributed to the O-type star*).

of secondary minimum (phase 0·5). In many plates the line was not even detected near primary minimum, and the relative intensities are probably fairly well represented by the two profiles of Fig. 5 derived from two plates with phases 0·993 and 0·508 respectively.

The strength of the line at secondary minimum is of the same order as that of the corresponding feature found in W spectra *, and it appears impossible to attribute it to the Wolf-Rayet component alone. If the identification of the line as due to *He*I is correct, there seems to be no escape from the conclusion that the material responsible for the displaced line is absorbing light from both components of the system and not that of the W star only. If this is the case, then it follows that the radius of the envelope responsible for *He*I absorption is larger than the distance between the two stars in the orbit. It would probably be more accurate to describe it as a shell than an envelope, since the helium in the inner portion nearer to the W star may be expected to be in an ionized state. Further, it appears that the greater intensity of the displaced line at secondary minimum will require an asymmetrical envelope with a greater amount of absorbing material on the side toward the O-type companion. Such a conclusion is in satisfactory accord with a similar explanation advanced to account for the asymmetrical form of the emission lines. The fact that the velocity from this line is only −400 km./sec., while *He*II bands are over 2000 km./sec. in width, suggests that the ejected material suffers a retardation of its velocity after leaving the surface of the star.

It is unfortunate that in the regions where other *He*I lines occur, conditions are

* Its displacement in ordinary W stars is much greater.

unfavourable for the observation of features of such small intensity. λ 4026 of *He*I is blended with λ 4025 of *He*II. In the visible region most of the observations were secured with fast emulsions of low contrast in conjunction with relatively low dispersion, so that displaced lines due to λ 5875 and λ 6678 of *He*I were not detected with certainty. As a consequence the above conclusion regarding the *He*I envelope rests upon observations of λ 4471 exclusively.

Appearance of Absorption on the Red Edges of Emission Bands.—Another feature which made its appearance on several microphotometer tracings of the spectrum of HD 193576 is an absorption border on the longer wave-length edge of certain of the emission bands. This feature was first observed in connection with the emission profile of $H\beta$ and its character is illustrated in Fig. 6. Later, similar but less well-marked features were detected in connection with the bands λ 4200 *He*II and λ 4101 $H+$*He*II $+N$III. It was noted that the absorption borders on the red edges of the bands were observed near primary minimum, and observations of the line $H\alpha$ at this phase were

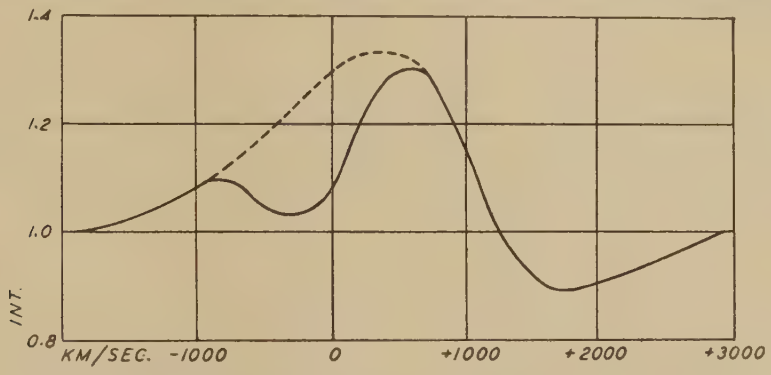

FIG. 6.—*Appearance of displaced absorption on the longer wave-length edge of $H\beta$ at phase* 0·036 (*near primary minimum*).

undertaken to see whether it showed similar effects. Such absorptions associated with $H\alpha$ were actually observed on a number of plates shortly after primary minimum.

The phenomena are somewhat elusive and some evidence of inherent variability in the W spectrum is suggested, but a review of all available data leaves little doubt that the observations are real.

Again, as in the case of the displaced absorption of λ 4471 of *He*I which appeared to the violet edge of its normal position, it appears that the absorptions to the red of the *He*II and $H+$*He*II bands finds its most logical explanation in terms of an expanding envelope of gases surrounding the W star. In this case the displaced absorption occurs only near primary minimum when the Wolf-Rayet component is between the O star and the observer. Since the feature appears on the red edge of the band, the absorbing material must be located between the two stars and the absorption must act only on the light of the O-type companion. From this it appears that the radius of the absorbing envelope (mainly *He*II) is smaller than the distance between the two stars. The fact that absorption features of comparable intensity are not found on the violet edges of the same bands may reasonably be attributed to the asymmetry of the envelope and its greater density on the side nearest the massive companion.

It is of some importance to point out that absorption of this kind, while it acts on the continuous spectrum of the bright O-type companion, may well show itself mainly as an apparent modification of the form or intensity of the emission band. While absorption lines of small intensity have been observed to the red of certain bands, the strongest absorption due to the W envelope is likely to occur over those wave-lengths where the emission is strongest, and a wide absorption line corresponding to the wave-lengths

covered by the emission band will be formed in the spectrum of the O-type star. The effect on the band intensity may be illustrated by the following calculation. Let I_w and I_o be the intensities of the undisturbed continuous spectra of the W- and O-type stars respectively. Let I_b be the intensity at any point in a rectangular band profile in the combined spectrum and let I_{bw} be the corresponding intensity in the spectrum of the Wolf-Rayet star alone.

Then

$$I_b = (I_{bw} + I_o)/(I_w + I_o). \tag{6}$$

For

$$I_{bw} = 10, \qquad I_w = 1 \quad \text{and} \quad I_o = 5,$$
$$I_b = 2{\cdot}5.$$

If by absorption the continuous spectrum of the O-type star over the wave-length range of the band is reduced to $0{\cdot}7I_o$, which affects the numerator only of equation (6), then I_b becomes

$$I_b = 2{\cdot}25.$$

Since I_b corresponds to continuous spectrum plus band, the figures for the band alone are 1·5 and 1·25 corresponding to a diminution of intensity 16·7 per cent. Since this corresponds to an absorption line of very moderate intensity, it is quite possible that the diminution in intensity of approximately 18 per cent. observed by Wilson in the equivalent width of λ 4686 at primary minimum is due to this cause.

Discussion of Conclusions

The results of this study have led to conclusions which are for the most part in satisfactory agreement with previous knowledge of O and W stars. The difference in brightness of 1·75 magnitudes between the two components of the binary corresponds closely in amount (2·0 magnitudes) and is in the same sense as the mean difference in magnitude between O and W stars as a class. The values of absolute magnitude and diameter are smaller than expected, but are not unreasonable in view of the known dispersion of these quantities and the uncertainty in the determination of the distance of the binary. The nature of the stratification of the envelope with the *He*II envelope of smaller diameter than that of *He*I is in accord with ionization theory and with observations of other emission-line stars and planetary nebulæ.

New information which may be of considerable importance to theories of Wolf-Rayet emission is found in the indication of retarded motion of ejected atoms of helium and in the conclusion that the *He*II envelope is smaller and the *He*I envelope is larger than the distance between the two stars. This is the first observational evidence pointing to a definite value for the radius of the emitting envelope of a Wolf-Rayet star. If the value for the radius of the star in Table V may be taken as approximately correct, then it appears that the *He*I shell is of the order of 30 times the stellar radius since the value of $a_1 + a_2$ from Wilson's spectrographic orbit is $38{\cdot}5\odot$. The *He*II shell is smaller, but the geometry of the system indicates that it must be several times that of the star in order to produce the absorption features to the red of the emission bands observed at primary eclipse.

The suggestion of an extended and semi-transparent photosphere with heavy darkening to the limb may also be of future importance in the development of the theory of Wolf-Rayet envelopes, and Kopal's study of the photometric elements now in progress should shed additional light on its diameter and absorbing qualities. If future discoveries ever brought to light a system like HD 193576 in which the eclipses were total or annular, then it should be possible to draw rather definite conclusions concerning the absorbing and emitting qualities of such an envelope.

The lack of agreement between the results of this study and those recently published by Wilson must be mentioned briefly in conclusion. In so far as the spectrophotometry of hydrogen lines and the classification of the O-type star is concerned the divergence in results is probably due in part to differences in the interpretation of complex spectral features as they appear on a microphotometer tracing. Wilson was apparently inclined to attribute the asymmetrical emission features of $H\gamma$ to the O-type star, whereas the present observations point to their identification as Wolf-Rayet bands whose asymmetrical features are closely correlated with those of λ 4686. Such a difference in interpretation may lead to values of equivalent width which would alter the results of the calculations of the relative brightness of the components of the binary. The method employed by the writer to calculate the relative brightness of the two stars is much less sensitive to observational errors than that employed by Wilson, but suffers from the defect that the assumptions as to the unmodified intensities of spectral features in the separate spectra cannot be checked directly. The agreement of the results with the mean values of relative absolute magnitude for other stars is, however, a very strong point in its favour.

Wilson's suggestion, based on the absence of a transit-time effect, that the origin of the Wolf-Rayet emission bands takes place close to the photosphere, is of course completely at variance with the writer's interpretation presented here and in previous papers on Wolf-Rayet stars.* Since Wilson has himself presented both sides of the question in considerable detail it is unnecessary to repeat the discussion here. One point, however, may be worthy of brief mention. Wilson's interesting and valuable discussion of the transit-time effect and the effect of occultation on the observed velocities was based on the assumption of complete spherical symmetry in the expanding envelope. According to the writer's interpretation of the spectrum of HD 193576, the presence of a tidal effect prevents this condition from being realized even approximately in the envelope of this star, and if this general idea be accepted, results for the magnitude of the transit-time effect would require modification in order to be applicable to the present problem. Even in the absence of any specific physical explanation of the changes in the emission profiles, the well-established asymmetry of the bands in the vicinity of the eclipses would appear to render the application of the theory of the transit-line effect somewhat uncertain. Further, it may be pointed out that results recently obtained by the writer for the star MWC 374 † suggest that absorption may play a very much more important rôle in the profiles of emission bands than had previously been supposed. If absorption of this kind acts to alter the profiles of Wolf-Rayet bands, the theoretical treatment of the occultation effect will also require revision.

In conclusion the writer wishes to acknowledge with thanks permission to make use of a number of plates taken by the Director, Dr. Pearce. Very sincere thanks are also extended to Dr. Zdenek Kopal for the opportunity to see his solution of the photometric light curve in advance of publication, and to Miss Elizabeth Walker who gave invaluable assistance in carrying out the spectrophotometric measurements on which this paper is based.

Dominion Astrophysical Observatory,
Victoria, B.C.:
1944 *March* 31.

* *P. Dom. Ap. O.*, 4, 272, 1929; 6, 95, 1933.
† *J.R.A.S. Can.*, 37, 241, 1943.

THE DIVISION ERRORS OF THE REVERSIBLE TRANSIT CIRCLE OF THE ROYAL OBSERVATORY, GREENWICH

Sir H. Spencer Jones, F.R.S., and R. T. Cullen

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Summary

The method of determination of the division errors of the fixed circle of the reversible transit circle of the Royal Observatory, Greenwich, is described. The circle has in effect two independent series of graduations, each 10′ apart; the microscopes are set so that adjacent microscopes read on the two series. This necessitates the determination of division errors over an arc of 120°. By mounting the auxiliary microscopes so that their positions can be varied within a range of 2°, the use of divided object-glasses is avoided and the determinations of the division errors can be arranged so as to reduce the accumulation of accidental errors. The error of each 5′ graduation over an arc of 120° is determined with a probable error less than ±0″·03.

The division errors have been analysed and found to consist of a combination of a recurrent $2\frac{1}{2}°$ variation, of periodic variations with periods of 15°, 30° and 60°, and of superposed accidental errors.

The Circle and Microscopes

The circles of the reversible transit circle of the Royal Observatory, Greenwich, are of glass, with a diameter of 28 inches, with an etched circle 24 inches in diameter of graduations at intervals of 5′. The instrument is provided with two circles, one fixed and the other movable. Each circle weighs 16 lbs. In order to eliminate flexure of the circles in their plane, each circle is provided round its rim with a series of twelve counterpoised lever supports; the lower six supports are operative in any position of the circle and the counterpoise weights are adjusted so that the operative supports just relieve the weight of the circle.

The etched graduations are all similar and of the same length, and are not numbered in any way. For convenience in setting the reading microscopes, six of the circle graduations, spaced at intervals of 60°, are marked with series of etched dots. A separate setting circle of 30 inches diameter, divided at 5′ intervals, with the degrees numbered, is provided and is read by a suitably placed low-power microscope, which serves as an index microscope. A separate low-power telescope is provided for setting the instrument.

Six micrometer microscopes, spaced at 60° intervals, are employed when the instrument is in normal use. One revolution of the micrometer corresponds to one minute of arc. The micrometer head is divided into 100 parts and is read by estimation to 0·001 revolutions of the screw. The sum of the readings of the six micrometer heads, when multiplied by 10, gives at once the mean reading expressed in seconds of arc. A toothed comb in the focal plane of each microscope enables the number of revolutions, or minutes of arc, to be added to the reading of the pointer microscope, to be read off quickly.

In the preliminary examination of the telescope at York it was found that there was an inequality of 1″·1 between consecutive 5′ intervals on the fixed circle. The circle was not divided on an automatic dividing engine, but by a special method of generation devised

by Messrs. Cooke, Troughton & Simms Ltd., which was expected to result in smaller division errors. The method enables graduations to intervals of 10′ to be conveniently made; the inequality referred to was introduced in the insertion of a second series of 10′ graduations, to bisect as nearly as possible the initial series. The fixed circle contains in effect two independent series of graduations at intervals of 10′, the second set being systematically displaced by about 0″·55 from the mid points of the first set. For the movable circle there was a similar displacement of amount 0″·33.

Such an inequality would be inconvenient in the application of corrections for runs and alternate division errors would be large. It was therefore decided to offset three of the microscopes, at 120° interval, by 5′. Thus, if the microscopes on the west pier are denoted by A, B, C, D, E, F (the sequence of lettering being in the direction of decreasing circle reading) and if A is set on the 0° 00′ graduation, E and C are on the graduations 120° 00′ and 240° 00′ respectively, whilst F, D and B are on the graduations 60° 05′, 180° 05′ and 300° 05′. Thus, in effect, we may regard the circles as having two independent series of graduations, each read by three microscopes spaced at intervals of 120°. The setting of the circle is always defined by the mean of the six microscopes; this mean is independent of the inequality of alternate divisions.

The adopted arrangement of the microscopes entails one disadvantage. The circle of a transit instrument is normally read by means of six microscopes, equally spaced at 60° apart. Division errors need only be determined relatively to the mean of the six microscopes; they therefore repeat after an arc of 60°, and the complete determination of the division errors entails the determination of the errors of 720 divisions. But with alternate microscopes offset by 5′, the division errors do not repeat until after an arc of 120°, so that the determination of the errors of 1440 graduations is needed. This is a serious addition to the heavy labour of determining the division errors.

The microscopes on the east pier are denoted by G, H, I, K, L, M; H, K, M are offset by 5′ so that the intervals HI, KL and MG are 60° 05′.

To reduce labour in the division error determinations, it is customary to use only two of the ordinary microscopes, A and D or G and K, at opposite ends of a diameter. These two microscopes on the Greenwich instrument are 180° 05′ apart, measuring from A or G in the direction of increasing reading. For the determination of the division errors two supplementary microscopes, in this case designated *a* and *c*, are introduced. These are normally 180° apart, but *c* was displaced by 5′, to make the interval from *a* to *c*, in the direction of increasing reading, 180° 05′.

Facilities are provided on the piers for mounting the supplementary microscopes at approximately 20° and 25° away from both A and D. For determination of the errors of single degrees, which are not a multiple of 5, it is customary to substitute divided object-glasses for the ordinary objectives of the auxiliary microscopes. The optical centres of the semi-lenses of each objective are displaced relatively to each other so that the images of two graduations, at a distance of 1° apart, are brought nearly into coincidence. The definition with the divided objective is necessarily somewhat inferior and the illumination is not so symmetrical as with the ordinary objective. Gill therefore suggested * that if another transit instrument were to be constructed, it would be well to provide arrangements for mounting the auxiliary microscopes at an interval of 24° from two of the main microscopes.

In the Greenwich reversible transit circle, the slot-pieces of the tubes of the auxiliary microscopes, used for clamping the microscopes to their bases on the pier, have been cut away so that the positions of these microscopes can be varied in their mountings by approximately 1° on either side of the mean position. Thus the auxiliary microscopes can be set at any desired position within the range of 19° to 21° or within the range 24° to 26° from A and D. This makes it possible for the division errors for each quarter-degree to be derived without the employment of divided object-glasses on the auxiliary

* *History and Description of the Cape Observatory*, p. 59, 1913.

microscopes, thereby avoiding the loss of accuracy that their use entails. At the same time the division error for any particular degree or quarter-degree is made to depend upon the errors of a greater number of divisions, determined in earlier stages of the work, thereby broadening the foundations of the determined results and minimising the spread of accidental errors.

In many investigations of division errors, the assumption has been made that the errors can be represented as a sum of periodic terms. This assumption can never be strictly accurate, though it may be approximately true when a circle is divided on an automatic dividing engine. It appeared *a priori* unlikely that it would be valid for a circle divided by an entirely different method. It was therefore decided that the errors of each 5′ division within an arc of 120° (each such error referring to the mean of the six divisions appearing under the six microscopes, spaced as already described) should be directly determined. The division errors of the fixed circle have been determined; the errors of the movable circle have not yet been investigated, but these can be determined differentially by measurements of the movable circle relative to the fixed circle.

In the observations for the determination of the division errors a number of different observers were employed, particularly in the earlier stages, in order to reduce to a minimum any effects of personality that might be introduced by the observers. The major part of the observations were made by Messrs. Cullen, Acton and Barton. Others participating, mainly in the earlier stages, were Mr. Greaves (until his appointment as Astronomer Royal for Scotland), Messrs. Symms, Wells, Blackwell and Terry.

Determination of Errors of 20° Divisions

The first stage of the work was to determine the errors of the graduations at 0°, 20°, 40°, 60°, 80° and 100°, which are adopted as primary divisions. The subsequent stages of the work are based upon the errors of these divisions, which therefore require to be determined with great accuracy. It was desired to determine the errors of each of the 5′ divisions with a probable error not exceeding $\pm 0''{\cdot}03$. For this purpose, the probable errors of the division errors of the primary graduations should be smaller than $\pm 0''{\cdot}010$.

The mean of the six divisions which include the division at 0° 00′ was adopted as the zero on which all results should be based and an arbitrary value of $0''{\cdot}50$ was adopted for this zero division error, in order to make all the division errors positive, and therefore additive to the circle readings. When the 0° 00′ division is under microscope A, the division 340° 00′ is under the auxiliary microscope *a*, when set at 20° from A, and the division 25° 00′ is under it when set at 25° from A.

The primary division errors are determined by using two of the ordinary microscopes, A and D on the west pier or G and K on the east pier, 180° 05′ apart, in conjunction with the auxiliary microscopes *a* and *c*, set at 20° from A and D or from G and K respectively.

With the instrument in the clamp W position, the instrument was set to pointer reading 270° 00′, so that the division 0° 00′ was under microscope A and the division 340° 00′ was under microscope *a*. The four microscopes A, *a*, D, *c*, were read. The instrument was then turned through 20°, to bring the 20° 00′ division under A and the 0° 00′ division under *a*. The four microscopes were again read. This process was repeated at 20° intervals until the initial setting was again obtained, thus forming a closed sequence. A similar series was then taken by a second observer. Two further series were obtained by the same two observers, the instrument being turned in the direction of decreasing circle reading. The four series, taken as near together in time as possible, form one set, the mean of the four series being taken.

If $20° + \xi$ is the angle between the zero points of the microscopes A and *a*, A_θ and a_θ are the readings of the microscopes A and *a*, when the $\theta°$ graduation is under the microscope A, and if d_θ is the division error of the single division θ, taken as additive to the

microscope reading, then

$$20^\circ + \xi = 20^\circ + (A_\theta + d_\theta) - (a_\theta + d_{\theta-20})$$

or

$$d_\theta - d_{\theta-20} = \xi + (a_\theta - A_\theta).$$

A similar relationship will apply to the mean division errors of pairs of opposite division marks, spaced 180° 05′ apart, if ξ denotes the amount by which the interval between the mean zero of the microscopes A and D and the mean zero of the microscopes *a* and *c* exceeds 20°.

But the division errors with which we are alone concerned refer to the means of the six microscopes. If, then, we consider the settings when θ has the values 0°, 120° and 240°, and form the mean of the three readings of microscope A and the three readings of microscope D, and also the mean of the three readings of microscope *a* and the three readings of microscope *c*, we obtain

$$d_0 - d_{340} = \xi + x_1,$$

where the *d*'s now relate to the mean of the six microscopes and x_1 is the difference between the mean readings of microscopes *a*, *c* and of microscopes A, D. But the division errors, for the mean of six microscopes, repeat after 120°. We have, therefore,

$$d_0 - d_{100} = \xi + x_1,$$

and similarly

$$\begin{aligned} d_{20} - d_0 &= \xi + x_2, \\ d_{40} - d_{20} &= \xi + x_3, \\ &\cdots\cdots \\ d_{100} - d_{80} &= \xi + x_6. \end{aligned}$$

It follows that

$$\xi = -\tfrac{1}{6}\Sigma_1^6 x.$$

Putting

$$d_0 = +0''\cdot50 = +0^r\cdot00833,$$

the primary division errors $d_{20}, d_{40}, \ldots d_{100}$ can be successively derived.

The determination of the primary division errors was based upon 96 sets of observations, 48 sets with the fixed circle west and 48 sets with the fixed circle east. In each position of the instrument, equal numbers of sets were obtained with increasing and decreasing circle readings respectively.

It may be noted that the differences between the errors of consecutive primary divisions, 0° and 20°, 20° and 40°, etc., are determined with equal precision, but that when the error of the division 0° is arbitrarily fixed the derived errors of the other primary divisions become slightly unequal. It can readily be shown that, with the method of derivation adopted, if ϵ_x is the probable error of measurement of the quantity x, and if ϵ_θ is the probable error of the determination of the error of the primary division θ°,

$$\begin{aligned} \epsilon_{20} = \epsilon_{100} &= \sqrt{\tfrac{5}{6}}\epsilon_x = 0\cdot913\epsilon_x, \\ \epsilon_{40} = \epsilon_{80} &= \sqrt{\tfrac{4}{3}}\epsilon_x = 1\cdot154\epsilon_x, \\ \epsilon_{60} &= \sqrt{\tfrac{3}{2}}\epsilon_x = 1\cdot225\epsilon_x. \end{aligned}$$

The value derived for a single value of ϵ_x is $\pm0^r\cdot00117 = \pm0''\cdot07$.

The observed and computed values of the probable errors of the primary division errors are as follows:—

Division	Obs. (Unit $0^r\cdot0001$)	Comp.
20°	±1·10	±1·07
40°	1·37	1·35
60°	1·55	1·43
80°	1·49	1·35
100°	0·86	1·07

The mean value is $\pm 0''.0076$; this was subsequently somewhat reduced by the incorporation of additional material derived in the course of the determination of the 5^c division errors as explained below.

The detailed results for the primary division errors are given for illustration in Table I.

TABLE I

Primary Division Errors of Fixed Circle

FIXED CIRCLE W

		20°	40°	60°	80°	100°
2 sets: Increasing Circle Reading.	1936 Oct. 13	+r.0088	+r.0064	+r.0071	+r.0087	+r.0057
	Oct. 16	88	43	55	90	78
2 sets: Decreasing Circle Reading.	Oct. 20	82	52	67	102	70
	Oct. 27	87	64	59	81	72
4 sets: Increasing Circle Reading.	1936 Oct. 13	96	64	72	88	60
	Oct. 17	101	68	86	104	64
	Oct. 20	95	60	62	63	55
	Oct. 24	92	57	65	88	68
4 sets: Decreasing Circle Reading.	1936 Oct. 14	106	77	77	92	78
	Oct. 19	104	83	85	104	69
	Oct. 23	96	74	80	107	74
	Oct. 27	87	56	63	98	68

FIXED CIRCLE E

2 sets: Increasing Circle Reading.	1936 Oct. 31	74	52	45	85	70
	Nov. 6	81	61	52	78	61
2 sets: Decreasing Circle Reading.	Nov. 9	92	58	57	81	71
	Nov. 13	91	58	53	89	65
4 sets: Increasing Circle Reading.	1936 Nov. 3	98	66	77	89	59
	Nov. 4	95	62	57	91	62
	Nov. 9	86	64	64	86	65
	Nov. 10	97	54	56	100	71
4 sets: Decreasing Circle Reading.	1936 Nov. 3	101	75	61	97	63
	Nov. 5	85	46	51	81	62
	Nov. 7	87	56	63	92	74
	Nov. 12	103	86	90	112	66
Mean		+·00922	+·00625	+·00653	+·00911	+·00668
Probable error of mean		±·00011	±·00014	±·00016	±·00015	±·00009

(Division error for 0° graduation adopted as +r.00833.)

Determination of Errors of 5° Divisions

The second stage of the work was the determination of the errors of the 5° divisions, not already included in the primary divisions. In this stage the microscopes *a* and *c* were set at 25° from A and D or from G and K respectively. With the division 0° 00′ under A or G, the division 25° 00′ was then under *a* and the division 205° 05′ was under *c*.

The microscope A (or G) was first set on the division 0° 00′ and the four microscopes A and D (or G and K), *a* and *c* were read. The instrument was then turned in one

direction through steps of 25°, the four microscopes being read at each step, until after five rotations of the instrument a closed sequence was completed. The mean of the settings on A at 0°, 240° (second rotation in the direction of increasing readings) and 120° (fourth rotation), with the corresponding settings on D at 180° 05′, 60° 05′ and 300° 05′ involves the division error for the 0° graduation. The mean of the corresponding settings on *a* and *c* involves the division error for the 25° graduation.

Thus we obtain the relation

$$d_{25} - d_0 = \xi - x_1,$$

where ξ denotes the amount by which the interval between the mean zero of microscopes A and D and the mean zero of the microscopes *a* and *c* exceeds 25°, and x_1 is the difference between the means, for the three particular settings involved, of the readings of microscopes *a*, *c* and of microscopes A, D. Similar equations are obtained for $d_{50} - d_{25}$, $d_{75} - d_{50}$, etc.

A complete set involves 24 such equations, the sum of whose left-hand sides is identically zero. These determine ξ, and then d_0, d_{25}, . . . can be obtained in succession. But, by proceeding in this way, no advantage is taken of the accurate determination of the primary division errors in the first stage of the work and, moreover, the weight with which the division errors are determined falls progressively from either end of the series towards its middle.

The results of the second stage were accordingly used in the first instance to provide a further determination of the primary division errors, which can be combined, with their appropriate relative weights, to obtain definitive values of the primary division errors.

The summation of the first four equations of the series gives

$$d_{100} - d_0 = 4\xi - (x_1 + x_2 + x_3 + x_4),$$

and of the whole 24 equations gives

$$\xi = \tfrac{1}{24}\Sigma_1^{24} x.$$

It follows that the probable error, ϵ, of the difference of two primary division errors is given by

$$\epsilon = \sqrt{\tfrac{10}{3}}\,\epsilon_x.$$

The corresponding value in the first stage of the work was $\sqrt{\tfrac{5}{6}}\,\epsilon_x$. Hence the probable error of the determination of the difference of two primary division errors from one set of observations in the second stage of the work is double the corresponding probable error from one set in the first stage. This is confirmed by the probable errors deduced from the internal accordance of the results in the two stages.

The first stage involved 96 complete sets of observations. In the second stage 36 complete sets were obtained. Hence the relative combining weights for the primary division errors are in the ratio $4 \times 96/36$ or 10·7 to 1. The results were actually combined with relative weights of 10 : 1. The increase in weight of the determination of the primary division errors, through the incorporation of the results of the second stage, is thus only about ten per cent., but is worth while. The mean probable error of the primary division error determinations is reduced to ±0″·0072. The adopted values of the primary division errors are given in Table II.

These adopted primary division errors were used as the basis for the derivation of the errors of the other 5° graduations. The 24 equations given by each set of observations by the 25° arcs were divided into six subgroups, each comprising four equations. Each subgroup involves two primary division errors and three of the additional 5° division errors. Adopting the primary division errors as given in Table II, the quantity ξ and

TABLE II

Combination of Primary Division Errors

	20° Arcs Weight 10	25° Arcs Weight 1	Adopted
$d_{100}-d_0$	$-^r\cdot00165$	$-^r\cdot00034$	$-^r\cdot00153$
$d_{80}-d_{100}$	$+\cdot00243$	$+\cdot00332$	$+\cdot00251$
$d_{60}-d_{80}$	$-\cdot00258$	$-\cdot00238$	$-\cdot00256$
$d_{40}-d_{80}$	$-\cdot00028$	$-\cdot00043$	$-\cdot00029$
$d_{20}-d_{40}$	$+\cdot00297$	$+\cdot00161$	$+\cdot00285$
d_0-d_{20}	$-\cdot00089$	$-\cdot00176$	$-\cdot00097$

the three intermediate 5° division errors are determined. Thus each 5° division errors is tied to two of the primary divisions errors, as follows:—

First	subgroup gives	d_{25}, d_{50}, d_{75}	tied to	d_0,	d_{100}.
Second	,, ,,	d_5, d_{30}, d_{55}	,, ,,	d_{100},	d_{80}.
Third	,, ,,	d_{10}, d_{35}, d_{105}	,, ,,	d_{80},	d_{60}.
Fourth	,, ,,	d_{15}, d_{85}, d_{110}	,, ,,	d_{60},	d_{40}.
Fifth	,, ,,	d_{65}, d_{90}, d_{115}	,, ,,	d_{40},	d_{20}.
Sixth	,, ,,	d_{45}, d_{70}, d_{95}	,, ,,	d_{20},	d_0.

From the first subgroup, it follows that

$$4\,d_{25}=3\,d_0+d_{100}-3x_1+x_2+x_3+x_4,$$
$$4\,d_{50}=2\,d_0+2\,d_{100}-2x_1-2x_2+2x_3+2x_4,$$
$$4\,d_{75}=d_0+3\,d_{100}-x_1-x_2-x_3+3x_4.$$

For a single set of observations, the probable errors of d_0, d_{100} are negligible compared with those of the x's. It is readily seen that the probable errors of the determinations of d_{25} and d_{75} are $\frac{1}{2}\sqrt{3}\epsilon_x$, and that of d_{50} is ϵ_x. From the internal accordance of the observations the derived mean probable error for the twelve 5° divisions adjacent to a primary division was $\pm0''\cdot0088$; the mean probable error for the 6 intermediate 5° divisions was $\pm0''\cdot0101$. The ratio of these probable errors is in close accordance with the expected value.

36 sets of observations for the 5° division errors were obtained as follows:—

Period of Observation		Position	Circle Reading	No. of Sets
1936	Nov. 20–Dec. 18	E	Increasing	12
	Nov. 23–Dec. 17	E	Decreasing	12
1936–7	Dec. 31–Jan. 18	W	Increasing	6
1937	Jan. 1–Jan. 16	W	Decreasing	6

The mean internal probable errors for the final values of the 5° division errors are $\pm0^r\cdot000146=\pm0''\cdot0088$ or $\pm0^r\cdot000168=\pm0''\cdot0101$, according to whether the division is, or is not, adjacent to a primary division. But this neglects the effect of the errors of the primary division error determinations; these are in the nature of systematic errors and affect the mean of any number of determinations of the 5° division errors to the same extent as they affect a single determination. The probable errors of the 5° division error determinations must therefore be increased to take into account the error introduced through being tied to the primary division errors. The effect depends somewhat upon the particular primary divisions that are involved. The average effect is to increase the probable errors from $\pm0''\cdot0088$ and $\pm0''\cdot0101$ to $\pm0''\cdot0101$ and $\pm0''\cdot0111$ respectively.

Determination of Errors of 1° Divisions

The third stage was the determination of the division errors of all the 1° divisions other than those derived in the first two stages. For this purpose advantage was taken of the

range of adjustment of the auxiliary microscopes. If, for instance, these microscopes are set at an angle of 24° from the two main microscopes, a series of 5 steps, each of 24°, makes 120°. Starting at division θ (which may be any one of the 5° divisions, whose error has already been determined), the division errors of the divisions $\theta+24°$, $\theta+48°$, $\theta+72°$ and $\theta+96°$ are derived in terms of the division error of θ (since the closing division $\theta+120°$ has the same error as θ).

The division errors of all the 1° divisions can be determined in this way. But if based entirely on 24° arcs, the division error of the commencing divisions affects systematically the division errors of the four degree divisions dependent upon it. Accordingly arcs of 26°, 19° and 21° have been used in addition. In each case, 5 steps gives an interval which is a multiple of 5°, so that four intermediate degree divisions are tied to two 5° divisions, whose errors have previously been determined.

By employing arcs of 19°, 21°, 24° and 26° it can readily be shown that the division errors of degree divisions represented by $5n\pm1$ are made to depend upon the errors of seven different 5° divisions, whilst those of the form $5n\pm2$ are made to depend upon the errors of five different 5° divisions. The precision of the determination of the degree division errors is thereby materially improved.

32 determinations of the error of each degree division of the form $5n\pm1$ or $5n\pm2$ were made as follows:—

Supplementary Microscopes a and c set to 24° 00′

Position	Period of Observation, 1937	Inst. turned	Position	Period of Observation, 1937	Inst. turned
W	Jan. 22–Jan. 26	Increasing Circle Reading	W	Jan. 27–Feb. 2	Decreasing Circle Reading
W	Feb. 3–Feb. 5	,,	W	Feb. 6–Feb. 9	,,
W	Feb. 9–Feb. 15	,,	W	Feb. 10–Feb. 13	,,
W	Feb. 16–Feb. 22	,,	W	Feb. 18–Feb. 23	,,

Supplementary Microscopes a and c set at 26° 00′

W	Feb. 25–Mar. 1	Increasing	W	Mar. 1–Mar. 5	Decreasing
W	Mar. 6–Mar. 9	,,	W	Mar. 11–Mar. 16	,,
W	Mar. 16–Mar. 20	,,	W	Mar. 22–Mar. 23	,,
W	Mar. 23–Mar. 24	,,	W	Mar. 25–Mar. 30	,,

Supplementary Microscopes a and c set at 19° 00′

E	May 20–May 21	Increasing	E	May 21–May 22	Decreasing
E	May 26	,,	E	May 24	,,
E	May 27–May 29	,,	E	May 31	,,
E	June 1–June 2	,,	E	June 2–June 3	,,

Supplementary Microscopes a and c set at 21° 00′

E	June 21–June 25	Increasing	E	June 28–June 29	Decreasing
E	June 29–July 1	,,	E	July 19–July 20	,,
E	July 21–Aug. 19	,,	E	Aug. 18–Aug. 20	,,
E	Aug. 25–Aug. 26	,,	E	Aug. 30–Aug. 31	,,

The observations were arranged in the following way. Using, for instance, the 24° arcs, microscope A was set successively on 0°, 24°, 48°, 72°, 96°, so that microscope *a* was on 24°, 48°, 72°, 96°, 120°. Similar subsets were made commencing with A on 60°, 120°, 180°, 240°, 300°. For each position the two main and the two auxiliary microscopes were read. Collecting the results for settings at 120° separation, this group of observations determines the errors of the divisions 24°, 48°, 72°, 96° and of 84°, 108°, 132° (12°), 156° (36°). Eleven further similar groups were obtained starting with A on 5°, 10°, 15° . . . 55°. Each group determines the errors of eight of the degree divisions,

the twelve groups together determining the errors of the 96 degree divisions, which are not of the form $5n$. Using the 19° arcs, microscope A is set successively on 19°, 38°, 57°, 76°, 95°, so that *a* is on 0°, 19°, 38°, 57°, 76°; similar subsets were made commencing with A on 79°, 139°, 199°, 259°, 319°. This group of observations determines the errors of the divisions 19°, 38°, 57°, 76° and of 79°, 98°, 117°, 136°. Eleven further similar groups were obtained starting with *a* on 5°, 10°, 15° . . . 55°. The observations for the 26° and 21° arcs were arranged similarly to those for the 24° and 19° arcs respectively.

If ϵ_x has the same meaning as previously, it is readily seen that the probable error of a determination of the error of a division of the form $5n \pm 1$ is $\sqrt{\frac{4}{5}}\epsilon_x$, and that of a division of the form $5n \pm 2$ is $\sqrt{\frac{6}{5}}\epsilon_x$. The probable error for each degree division error was determined from the internal accordance of the 32 sets of observations. The mean probable error for a single set for the 48 divisions of form $5n \pm 1$ is found to be $\pm 0^r{\cdot}000994$; the corresponding mean for the 48 divisions of form $5n \pm 2$ is found to be $\pm 0^r{\cdot}001182$. The inferred values of ϵ_x are $\pm 0^r{\cdot}00111$ and $\pm 0^r{\cdot}00108$ respectively, which are in close agreement with each other, and with the values derived in the course of the other stages of the division error determinations.

The probable error of the adopted division errors of the 1° divisions are $\pm 0^r{\cdot}000176$ or $\pm 0''{\cdot}011$ for the degree divisions represented by $5n \pm 1$, and $\pm 0^r{\cdot}000209$ or $\pm 0''{\cdot}013$ for those represented by $5n \pm 2$. The dependence on the 5° division errors is not of a systematic nature and is included in these probable errors, based on the accordance of the various determinations.

Determination of Errors of 15′ Divisions

The fourth stage of the investigation was the determination of the 1/4 degree (15′, 30′ and 45′) division errors. These also were determined with the supplementary microscopes *a* and *c*, together with the main microscopes A and D on the west pier or G and K on the east pier, by a method similar to that employed for the degree division errors. In order to broaden the basis of the determination of these errors, and to obviate the errors of the divisions to which the quarter-degree divisions are tied having a systematic effect, arcs of 19° 15′, 19° 45′, 20° 15′, 20° 45′, 24° 15′, 24° 45′, 25° 15′ and 25° 45′ were used. With each of these arcs, four steps makes an exact number of degrees, so that three of the quarter-degree divisions can be tied to two of the degree divisions.

It can readily be shown that by using the eight different arcs, any division of the form $n \pm 1/4$ is tied on to 14 different degree divisions and any division of the form $n \pm \frac{1}{2}$ is tied on to 16 different degree divisions.

Eight determinations of the errors of each of the 15′, 30′ and 45′ divisions were made, as follows:—

Position	Period of Observation	Instrument turned	Arc used
E	1937 Sept. 20–Oct. 1	Increasing Circle Readings	20° 45′
E	1937 Oct. 7–Oct. 13	Decreasing ,, ,,	20° 15′
E	1937 Oct. 19–Nov. 3	Increasing ,, ,,	19° 45′
E	1937 Nov. 5–Nov. 22	Decreasing ,, ,,	19° 15′
W	1937–8 Dec. 4–Jan. 5	Increasing ,, ,,	24° 15′
W	1938 Jan. 10–Jan. 22	Decreasing ,, ,,	24° 45′
W	1938 Jan. 27–Mar. 1	Increasing ,, ,,	25° 15′
W	1938 Mar. 8–Mar. 26	Decreasing ,, ,,	25° 45′

A complete set of observations was arranged as follows. Using, for instance, the 20° 45′ arcs, microscope A was set successively on 20° 45′, 41° 30′, 62° 15′ and 83° 00′, so that microscope *a* was on 0° 00′, 20° 45′, 41° 30′ and 62° 15′. Similar subsets were made commencing with *a* on 60°, 120°, 180°, 240°, 300°. This set of observations determines the errors of the 6 divisions 20° 45′, 41° 30′, 62° 15′ and 80° 45′, 101° 30′, 2° 15′. Similar sets were obtained starting with microscope *a* on the divisions 1°, 2°,

3° . . . 59°. The 60 groups together determine the errors of the 360 quarter-degree (15′, 30′ and 45′) graduations.

It is easily shown that the probable error of one determination of the error of any 15′ or 45′ division is $\sqrt{\frac{3}{4}}\epsilon_x$, and of the error of any 30′ division is ϵ_x. The mean probable error of a single determination derived from the internal accordance of the eight separate determinations is, for the 15′ and 45′ divisions, ±0ʳ·000940 and, for the 30′ divisions, is ±0ʳ·00110. The inferred values for ϵ_x are ±0ʳ·00108 and ±0ʳ·00110 respectively, which are in satisfactory accordance with each other and with other determinations.

The value of the probable error of the derived values for the 15′ and 45′ division errors is ±0ʳ·000332 or ±0″·020, and for the 30′ division error is ±0ʳ·000389 or ±0″·023. These values include the effect of the errors of the 1° divisions to which the quarter-degree divisions are tied.

Determination of Errors of 5′ Divisions

The fifth and final stage of the investigation was the determination of the errors of the remaining divisions, *i.e.* of the 5′, 10′, 20′, 25′, 35′, 40′, 50′ and 55′ divisions of each degree. In this final stage the supplementary microscopes were not used, but the six ordinary microscopes were employed and were read for each setting of the instrument, which was moved on progressively by means of the slow motion through angles of 5′. Bringing each 5′ division in succession to the zero of microscope A, a direct measure of the interval to the next 5′ division is obtained on each of the six microscopes by setting on the divisions at both the 0 and 5 ends of the comb; the means for the six microscopes are formed. The 5′ divisions, between the quarter-degree divisions, are thus tied to the adjacent quarter-degree divisions.

If $5' + \xi$ denotes the mean interval (allowing for the effect of runs), the equations connecting the errors d_1, d_2 of the intermediate divisions with those of the adjacent quarter-degree divisions D, D' are of the form

$$\begin{aligned} D - d_1 &= -\xi + x_1, \\ d_1 - d_2 &= -\xi + x_2, \\ d_2 - D' &= -\xi + x_3, \end{aligned}$$

where the x's denote the differences between the mean micrometer readings for the two settings.

These equations give

$$\begin{aligned} 3d_1 &= 2D + D' - 2x_1 + x_2 + x_3, \\ 3d_2 &= D + 2D' - x_1 - x_2 + 2x_3. \end{aligned}$$

Since in these observations any given 5′ division is always tied to the same two quarter-degree divisions, any error in the determinations of the division errors for these divisions will affect equally all the determinations of the error of this particular 5′ division.

Six determinations of the 5′ division errors were made as follows:—

Arc of Circle under Observation	Position	Period of Observation
0° 00′ to 120° 00′	E	1938 Oct. 18–1938 Dec. 23
120° 00′ ,, 240° 00′	E	1938 Dec. 28–1939 May 8
240° 00′ ,, 360° 00′	E	1939 May 8–1939 Dec. 19
0° 00′ ,, 120° 00′	W	1939 Jan. 5–1939 Feb. 27
120° 00′ ,, 240° 00′	W	1939 May 10–1939 Aug. 10
240° 00′ ,, 360° 00′	W	1939 Aug. 11–1940 July 1

The individual determinations for 108 divisions, spaced at intervals of 1° 05′ or 1° 10′, were used to give a mean value of the probable error of a single determination. This was found to be ±0″·062. The adopted errors, based on the mean of 6 sets, therefore have a probable error of ±0″·025, given by the internal accordance of the

determinations. This value needs to be increased to allow for the uncertainty of the division errors of the divisions to which they are tied; these average ±0″·019. The expressions for d_1, d_2 given above show how the quarter-degree division errors affect the determinations of the 5′ division errors tied to them. Their effect is to increase the probable error of the 5′ division errors from ±0″·025 to ±0″·029.

Summary of Determinations

The successive stages of the investigation can be summarised as follows:—

Stage	Division Errors Determined	Adopted Arcs	Results based on	No. of Determinations	No. of Microscope Readings	Average Probable Error of Result
1	20°	20°	Zero division	96	7,296	±0″·007
2a	20°	25°	,,	36		
2	5°	25°	Two 20° divisions	36	10,368	±0″·009
3	1°	19°, 21°, 24°, 26°	Five or seven 5° divisions	32	46,080	±0″·012
4	¼°	19° 15′, 19° 45′, 20° 15′, 20° 45′, 24° 15′, 24° 45′, 25° 15′, 25° 45′	14 or 16 1° divisions	8	46,080	±0″·021
5	5′	...	Two ¼° divisions	6	103,680	±0″·029

It will be seen that the complete investigation involved 213,504 microscope readings.

The division errors for each division are given in Table II. The values given are all additive to the circle readings, the division error for the zero division having been arbitrarily adopted so as to make the errors all of the same sign. The extreme range in the errors is 0″·75.

Analysis of Division Errors

A casual glance at the Table is sufficient to show that there is a marked tendency for large or small entries to recur after 2° 30′.

To investigate the 2½° variation, the division errors in Table II were divided into periods of 2½°, and the 12 periods in each of the intervals 0°–30°, 30°–60°, 60°–90° and 90°–120° were summed. The residuals from the mean value for each group and the general mean are given in Table III.

The general similarity in the 2½° variation as derived from each of the four groups of 30° will be noted. The variation is an irregular one, and cannot be represented as a periodic variation or as the sum of periodic variations. The results show that the determination of the division error of each 5′ graduation was necessary; the rapid changes in the division errors from 1° 10′ to 1° 20′ and from 2° 10′ to 2° 20′ may be noted.

The differences between the mean values of the division errors over the four arcs of 30°, given at the foot of the table, are caused by errors in the dividing of a periodic nature, referred to below.

The mean values of the 2½° variation, taken over arcs of 30°, show differences which are only partially due to the accidental errors of the determinations of the division errors. There are some quasi-systematic differences, which cannot be accounted for by errors of observation. When, for instance, the means of the 5′ and 10′ division errors over the 2½° are formed, differences are shown which are much greater than the actual errors of observation would permit. These mean values are given at the foot of the table; the means for 0° to 30° and for 90° to 120° are in close agreement, as are also the means for 30° to 60° and 60° to 90°, but the two sets of means are discordant. Thus though a

TABLE II

Reversible Transit Circle *Unit* 0″·01

Division Errors of the Fixed Circle

Mic. A	00′	05′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′
°													
0	50	32	68	50	54	51	53	42	67	54	69	69	47
1	47	32	36	40	64	44	48	27	47	22	45	27	47
2	47	49	46	32	30	40	58	61	78	64	66	59	56
3	56	38	61	45	71	61	67	54	39	48	59	56	52
4	52	38	50	35	26	38	45	48	49	43	40	40	59
5	59	57	64	62	59	64	51	53	57	61	62	58	59
6	59	38	23	51	77	67	71	57	61	46	56	46	45
7	45	35	52	37	48	44	61	64	76	73	69	49	43
8	43	54	64	63	79	78	58	50	43	60	68	57	53
9	53	57	50	36	50	49	53	52	63	51	54	59	58
10	58	66	84	62	72	58	53	54	66	64	80	82	59
11	59	61	31	63	64	64	43	53	50	43	41	56	61
12	61	52	52	44	29	43	60	72	82	62	69	56	53
13	53	53	64	66	68	59	56	48	42	53	61	60	47
14	47	48	50	50	47	50	44	44	46	46	32	48	56
15	56	70	73	68	52	62	57	60	66	66	66	73	56
16	56	55	52	56	56	46	40	40	34	40	36	46	59
17	59	38	51	40	24	38	53	65	74	64	68	46	41
18	41	48	58	57	60	57	56	47	45	42	69	51	45
19	45	33	40	39	41	40	49	45	55	54	31	42	56
20	56	64	63	71	56	45	52	46	60	56	59	55	60
21	60	47	39	49	64	61	50	50	41	42	31	36	55
22	55	42	53	47	34	45	60	64	82	59	61	58	58
23	58	59	70	60	78	54	60	43	48	42	66	61	39
24	39	35	39	46	46	38	62	67	71	54	33	43	56
25	56	55	71	57	62	59	58	62	61	59	67	49	60
26	60	42	44	42	72	55	45	39	34	38	37	42	47
27	47	38	47	41	30	48	63	58	82	53	60	61	66
28	66	57	69	48	66	39	49	38	50	52	65	54	38
29	38	36	42	23	30	43	45	59	54	38	34	40	48
30	48	64	62	49	58	54	47	57	67	58	77	56	58
31	58	44	26	43	62	47	42	36	44	30	32	36	33
32	33	36	33	38	32	50	55	66	79	59	56	52	58
33	58	55	61	54	66	54	55	42	34	40	57	68	40
34	40	43	32	33	36	32	35	44	50	25	34	35	50
35	50	56	59	53	42	46	44	39	40	47	70	45	55
36	55	42	34	49	64	50	49	41	42	43	35	36	44
37	44	40	51	33	24	37	45	54	56	58	52	50	38
38	38	49	50	52	59	61	49	54	38	44	46	47	32
39	32	40	39	27	18	42	38	60	43	41	22	42	39
40	39	44	53	53	48	44	33	46	36	44	62	52	43
41	43	41	20	44	48	37	35	40	39	26	25	32	38
42	38	39	48	28	19	44	36	51	50	57	37	38	29
43	29	43	52	45	53	44	40	46	15	39	41	47	16
44	16	38	41	26	30	29	32	39	26	19	19	36	33

TABLE II—*continued*

Mic. A	00′	05′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′
°													
45	33	40	53	47	41	40	26	33	31	46	38	52	34
46	34	48	22	36	41	44	28	25	18	25	18	33	38
47	38	40	44	32	9	28	34	48	55	62	44	46	32
48	32	38	36	51	41	58	43	34	22	39	44	40	25
49	25	29	24	22	19	36	40	40	51	42	17	44	47
50	47	55	60	64	42	41	28	49	40	48	47	55	36
51	36	34	25	37	51	44	36	30	22	28	35	44	41
52	41	39	40	33	18	26	52	55	57	53	50	55	33
53	33	53	48	60	50	56	54	48	43	46	52	44	38
54	38	47	33	32	28	46	43	52	50	43	29	46	46
55	46	61	56	62	50	50	43	44	48	52	54	58	53
56	53	44	47	48	51	51	41	53	28	52	25	44	44
57	44	58	51	49	32	51	45	81	62	55	46	57	53
58	53	51	43	58	49	54	55	34	44	45	55	58	44
59	44	45	17	39	14	29	36	53	52	45	27	37	41
60	41	63	54	70	53	48	42	70	60	60	62	60	56
61	56	52	36	50	57	54	32	57	22	44	25	45	43
62	43	57	44	40	22	46	65	75	71	71	53	60	44
63	44	57	40	60	54	64	52	42	40	53	53	40	42
64	42	46	30	39	30	40	53	55	48	54	32	50	47
65	47	62	67	62	61	45	51	61	54	53	49	57	49
66	49	38	31	48	67	73	47	62	52	56	55	72	47
67	47	61	50	47	23	42	46	61	73	67	60	61	59
68	59	66	66	58	68	68	68	46	54	51	63	64	53
69	53	53	40	35	32	56	61	46	56	60	44	45	58
70	58	65	72	70	57	67	56	56	49	70	69	64	74
71	74	45	53	47	75	56	53	54	51	35	34	42	50
72	50	58	56	40	32	46	55	75	82	67	54	59	49
73	49	56	52	59	51	60	59	53	33	49	66	47	40
74	40	54	48	40	36	47	52	44	51	36	35	45	52
75	52	59	84	64	67	65	58	54	57	69	80	63	73
76	73	48	36	52	61	51	47	40	40	34	24	31	39
77	39	59	50	36	36	37	56	64	63	68	58	60	45
78	45	50	54	57	67	62	55	66	39	59	53	53	37
79	37	41	35	35	34	46	38	54	67	46	28	49	56
80	56	50	71	60	51	54	48	56	49	70	67	69	53
81	53	56	47	52	65	52	49	43	34	30	37	42	47
82	47	64	49	54	47	52	61	59	64	76	53	54	50
83	50	62	56	73	59	74	64	63	35	58	64	57	45
84	45	44	53	37	32	54	49	58	61	65	48	48	61
85	61	68	68	73	55	60	58	59	52	73	61	67	57
86	57	68	38	50	59	63	49	42	29	37	43	44	40
87	40	44	46	43	30	42	55	72	64	71	46	57	52
88	52	59	54	72	46	66	48	51	41	55	64	53	38
89	38	40	32	43	24	31	51	52	63	43	32	37	47
90	47	59	60	71	56	57	52	49	59	72	63	69	59
91	59	50	35	45	48	60	45	48	33	38	35	39	43
92	43	46	43	27	29	41	51	69	76	71	58	53	45
93	45	59	49	59	51	57	52	39	33	34	57	56	55
94	55	40	37	32	23	40	36	43	34	41	31	47	46

TABLE II—*continued*

Mic. A	00′	05′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′
°													
95	46	53	52	49	59	42	41	49	46	55	46	57	44
96	44	40	37	48	64	49	42	43	34	31	27	29	32
97	32	50	40	34	21	37	50	52	59	51	53	45	40
98	40	47	56	51	64	52	55	46	33	42	55	35	40
99	40	37	44	40	27	27	48	42	57	27	31	27	41
100	41	42	50	50	53	39	34	20	38	50	55	64	53
101	53	27	29	29	50	45	34	29	36	29	13	22	38
102	38	40	31	32	23	28	51	56	74	52	52	37	35
103	35	36	39	51	48	49	51	39	27	28	60	36	40
104	40	20	30	34	14	29	38	35	31	24	14	27	40
105	40	61	56	48	41	29	39	30	45	37	50	38	55
106	55	34	34	35	49	27	22	11	22	13	13	16	43
107	43	39	35	29	14	18	44	54	60	53	57	42	40
108	40	45	54	35	54	52	46	32	18	36	50	40	40
109	40	22	34	28	26	26	38	43	46	30	28	34	47
110	47	62	75	58	62	55	42	30	37	30	52	40	43
111	43	25	30	43	54	48	40	24	36	32	33	27	35
112	35	52	46	28	26	47	39	55	60	53	55	40	49
113	49	38	50	40	54	53	62	37	32	55	54	44	41
114	41	46	42	27	33	24	45	49	62	41	28	38	57
115	57	57	68	42	52	49	54	56	56	62	78	66	53
116	53	41	37	54	66	51	47	40	50	28	46	34	49
117	49	47	54	33	36	38	70	44	68	52	47	46	58
118	58	60	60	46	67	65	52	46	23	57	60	41	39
119	39	47	51	25	37	36	51	44	51	40	25	36	50

recurrent variation with a period of $2\frac{1}{2}°$ is clearly shown all round the circle, it is slightly variable in amount through the arc of 120°. This admits of a simple explanation, which will be mentioned presently.

The mean $2\frac{1}{2}°$ variation, given in the last column of Table III, was subtracted from the determined division errors and the residuals so obtained were meaned for each degree, in order to examine for the presence of errors of a periodic nature.

The mean errors per degree, obtained in this way, repeated closely after 60° and could be well represented as the sum of three periodic terms, with periods of 15°, 30° and 60°. By harmonic analysis, the periodic terms were found to be:

$$-0''{\cdot}017 \sin 24\theta - 0''{\cdot}014 \cos 24\theta + 0''{\cdot}006 \sin 12\theta + 0''{\cdot}022 \cos 12\theta + 0''{\cdot}077 \sin 6\theta - 0''{\cdot}001 \cos 6\theta,$$

where θ is measured from 0°.

The observed and computed values are given in Table IV. There is close general agreement between the observed and computed values, showing that on the whole the mean errors are satisfactorily represented by the periodic terms. Individual observed values deviate from the computed values by amounts in excess of what can be attributed to systematic errors. The deviations for the two 30° arcs, 60° apart, tend to be in the same direction; compare, *e.g.*, the observed values for 6° and 15° in the second and third columns and for 5° and 10° in the fifth and sixth columns. The mean values at the foot of the table account for the differences in the mean values of the division errors shown at the foot of Table III.

TABLE III

$2\frac{1}{2}°$ *Variation*

(Unit 0″·01)

	0°–30°	30°–60°	60°–90°	90°–120°	0°–120° Mean
° ′					
0 00	+ 4·9	+ 1·1	+ 1·9	+ 5·3	+ 3·3
05	+ 8·1	+13·2	+11·9	+12·0	+11·3
10	+22·1	+15·4	+16·9	+19·9	+18·6
15	+ 9·5	+12·9	+15·7	+10·9	+12·3
20	+ 9·7	+ 4·1	+ 3·2	+10·5	+ 6·9
25	+ 3·1	+ 4·7	+ 5·0	+ 1·2	+ 3·5
30	+ 0·8	− 4·4	− 1·5	+ 0·8	− 1·0
35	− 0·4	+ 3·3	+ 6·3	0·0	+ 1·8
40	+11·0	+ 2·9	+ 1·1	+ 5·8	+ 5·2
45	+ 5·7	+ 8·2	+12·0	+ 5·7	+ 7·9
50	+16·1	+12·4	+ 8·6	+13·5	+12·7
55	+ 8·6	+10·7	+12·0	+11·9	+10·8
1 00	+ 4·6	+ 4·8	+ 6·3	+ 8·8	+ 6·1
05	− 6·3	− 0·5	− 0·2	− 5·3	− 3·1
10	−11·6	−12·3	−12·2	−12·6	−12·2
15	− 2·8	− 0·6	− 0·5	− 1·1	− 1·2
20	+12·8	+ 7·9	+ 9·7	+12·3	+10·7
25	+ 3·7	+ 5·0	+ 2·7	+ 1·0	+ 3·1
30	− 5·0	− 7·6	− 8·2	− 2·9	− 5·9
35	− 9·9	− 4·2	− 4·5	− 9·4	− 7·0
40	− 7·8	−11·5	−13·7	− 5·9	− 9·7
45	−14·3	−11·2	−13·7	−13·5	−13·2
50	−12·1	−16·8	−18·7	−16·0	−15·9
55	−10·0	− 6·5	− 6·7	−14·2	− 9·3
2 00	− 1·6	− 4·3	− 5·0	− 2·0	− 3·2
05	− 5·2	+ 2·0	+ 1·8	+ 0·9	− 0·1
10	+ 0·7	+ 1·8	+ 0·9	+ 0·9	+ 1·1
15	− 8·7	− 7·4	− 5·5	−11·1	− 8·2
20	−17·7	−19·6	−18·4	−17·8	−18·4
25	− 8·4	− 3·4	− 7·6	− 8·5	− 7·0
Mean value of division error	52·6	43·1	52·5	43·3	
Mean of 10′ divisions	+ 1·8	− 1·7	− 1·9	+ 1·4	
„ „ 5′ „	− 1·8	+ 1·7	+ 1·9	− 1·4	

Thus, while the division errors can in the main be represented by the combination of a recurrent irregular variation having a period of $2\frac{1}{2}°$, with simple harmonic periodic terms having periods of 15°, 30° and 60°, there are small erratic errors superposed upon these. The combination of the harmonic terms with the $2\frac{1}{2}°$ periodicity does not represent the observed division errors within the limits of the observational errors.

That the residual errors after subtracting the $2\frac{1}{2}°$ variation and the three periodic terms are not merely errors of observation can be made evident in the following manner. The residual error for each graduation was derived and the sums per degree of the 10′ (00′, 10′, 20′, etc.) graduations and of the 5′ (05′, 15′, 25′, etc.) graduations were separately formed. These sums were meaned for 5° intervals, centred at 0°, 5°, 10°, etc. The mean values so obtained are given in Table V.

It will be noted that the 10′ sums in the second column of the above table are in close agreement with the 5′ sums in the last column, and that the 5′ sums in the third column

TABLE IV

Mean Residual Division Errors per Degree after Subtracting $2\frac{1}{2}°$ Variation

(Unit 0″·01)

θ 0° 30° 60° 90° +	0°–29°	60°–89°	Computed 0°–29° 60°–89°	30°–59°	90°–119°	Computed 30°– 59° 90°–119°
0	− 0·7	+ 1·2	+0·9	+ 2·4	+ 3·8	+ 0·3
1	− 3·3	+ 0·9	+1·3	− 1·7	+ 1·3	− 0·9
2	+ 2·9	+ 4·3	+2·0	− 0·5	+ 1·0	− 1·8
3	+ 3·1	− 1·6	+3·0	+ 2·2	− 2·2	− 2·4
4	+ 2·0	+ 3·2	+4·1	− 3·4	− 1·8	− 2·8
5	+ 3·2	+ 0·1	+5·3	− 6·4	− 6·1	− 3·0
6	+11·1	+10·9	+6·5	+ 1·8	− 2·6	− 3·1
7	+ 4·8	+ 3·6	+7·4	− 4·3	− 5·9	− 3·4
8	+ 8·2	+ 9·4	+8·0	− 2·6	− 3·5	− 3·8
9	+10·6	+ 8·4	+8·2	− 3·0	− 2·8	− 4·6
10	+10·9	+ 7·1	+8·0	− 9·5	−11·0	− 5·6
11	+ 9·1	+ 8·3	+7·4	− 7·4	−10·2	− 6·9
12	+ 8·1	+ 6·6	+6·5	− 9·2	− 6·8	− 8·3
13	+ 5·4	+ 1·3	+5·5	−10·3	− 9·9	− 9·7
14	+ 6·0	+ 4·0	+4·5	−10·8	−12·3	−10·9
15	+ 8·4	+ 8·7	+3·7	−15·7	−12·8	−11·7
16	+ 3·2	+ 1·5	+3·2	−12·3	−15·7	−12·1
17	+ 2·1	+ 2·6	+3·0	− 9·6	− 8·9	−11·9
18	+ 1·1	+ 3·5	+3·3	−11·7	−16·5	−11·1
19	+ 2·8	+ 2·5	+3·9	− 7·6	− 7·1	− 9·9
20	+ 1·2	+ 2·7	+4·7	− 8·0	− 6·5	− 8·3
21	+ 4·2	+ 3·4	+5·5	− 8·1	− 7·0	− 6·5
22	+ 5·4	+ 7·1	+6·3	− 6·3	− 4·9	− 4·7
23	+ 6·8	+ 8·1	+6·7	− 2·6	− 4·2	− 3·1
24	+ 7·8	+ 8·7	+6·8	+ 0·6	− 0·3	− 1·8
25	+ 4·0	+ 7·2	+6·4	− 3·7	+ 2·4	− 0·7
26	+ 2·6	+ 5·0	+5·6	+ 1·5	+ 2·3	− 0·1
27	+ 2·7	+ 1·3	+4·4	+ 3·0	− 0·9	+ 0·3
28	+ 2·9	+ 3·6	+3·1	− 1·6	+ 1·4	+ 0·5
29	+ 0·1	+ 0·5	+1·6	− 3·5	+ 0·2	+ 0·7
Means	+ 4·6	+ 4·5	+4·9	− 4·9	− 4·7	− 4·9

are in close agreement with the 10′ sums in the fifth column. The closeness of this agreement indicates that the residual errors are not merely accidental.

The explanation of the agreement is as follows. It has been mentioned that the circle contains in effect two independent series of 10′ graduations and that the microscopes are arranged so that three microscopes 120° apart read one series of graduations and the three intermediate microscopes, 60° 05′ away, read the other series.

If $2f_1(\theta)$, $2f_2(\theta)$ represent the division errors for the two series of graduations respectively, as given by means of three microscopes, then at $\theta°$ the observed division error

TABLE V

Means for 5° Intervals of Sums of Six 10′ and 5′ Residual Errors

(Unit 0″·01)

	Graduations			Graduations	
	′ 10	′ 5		′ 10	′ 5
°			°		
0	+16	−27	60	−27	+20
5	+20	−15	65	−24	+10
10	+16	+18	70	+13	+ 6
15	+ 9	+11	75	+ 4	0
20	+ 3	−14	80	−10	+ 9
25	+16	−19	85	−12	+25
30	+14	−11	90	− 3	+23
35	+10	− 5	95	−13	− 2
40	−23	+ 2	100	+ 3	−27
45	−31	+ 3	105	− 1	−28
50	−22	+ 3	110	+11	−16
55	−17	+25	115	+26	−18

can be represented by $f_1(\theta)+f_2(\theta+60°)$; at $\theta°+5'$ the observed division error can be represented by $f_1(\theta+60°\ 10')+f_2(\theta)$. Thus the means over 5° in Table V can be closely represented as follows:—

For the 10′ graduations by $f_1(\theta)+f_2(\theta+60°)$,
„ „ 5′ „ „ $f_1(\theta+60°)+f_2(\theta)$.

When θ is increased by 60°, since $f_1(\theta)$ and $f_2(\theta)$ both repeat after 120°, it is clear that the mean for the 5′ graduations for $\theta+60°$ should be equal to the mean for the 10′ graduations for θ, and the mean for the 10′ graduations for $\theta+60°$ should be equal to the mean for the 5′ graduations for θ.

Referring to Table III, the mean of the 10′ divisions given at the foot of any of the four columns should equal the mean of the 5′ divisions for the column corresponding to an angle differing by 60°. The values are, in fact, in close agreement in the manner to be expected.

The accuracy of the determination of the division errors and the fact that the residual errors are not accidental can be shown in a different manner as follows. The sums of the residual errors for the six 10′ graduations in a degree and also for the six 5′ graduations were formed and the latter sum subtracted from the former. The differences so obtained are given for $\theta=0°$ to 59° in Table VI (denoted by A) and next to them are given the corresponding differences for $\theta+60°$, *with their signs changed* (denoted by B).

The sums represented by A depend on

$$f_1(\theta)+f_2(\theta+60°)-f_1(\theta+60°)-f_2(\theta).$$

When θ is increased by 60°, this function assumes the same value with reversed sign. The equality of the quantities denoted by A and B is therefore to be expected, apart from the effect of the accidental errors of the determinations of the division errors. The close agreement of these sums, derived from the observed division errors, may be noted. Attention may be drawn to the rapid changes from 4° to 6°, from 10° to 12°, from 57° to 59° and elsewhere in the table, and to the long sequences without change of sign, again showing that the residual errors are not accidental.

The probable error of the difference (A − B) is ±0″·074, corresponding to a probable error for A or B of ±0″·052. This enables an independent estimate of the probable error of the 05′, 10′, 20′, 25′, 35′, 40′, 50′ and 55′ division errors in every degree to be

TABLE VI

Differences of Sums per Degree of 10′ and 5′ Graduations

(Unit 0″·01)

°	A	B	°	A	B	°	A	B
0	+65	+56	20	+11	+12	40	−11	− 8
1	+92	+76	21	− 1	− 8	41	−14	−30
2	+23	+48	22	+33	+34	42	−27	−26
3	+50	+35	23	+59	+60	43	−37	−18
4	+29	+39	24	+15	0	44	−16	− 5
5	0	+ 8	25	+36	+43	45	−35	−29
6	+40	+50	26	+31	+32	46	−53	−55
7	+53	+37	27	+33	+46	47	−29	−22
8	− 9	−23	28	+75	+54	48	−43	−31
9	+ 7	+ 1	29	+13	− 2	49	−28	−38
10	+29	+29	30	+22	+41	50	−41	−43
11	−55	−58	31	+25	+29	51	−14	−35
12	+16	+15	32	−10	+ 4	52	0	+10
13	+ 2	+15	33	+16	+19	53	−28	−33
14	−13	− 3	34	+23	+19	54	−37	−34
15	−28	−25	35	+21	+13	55	−27	−36
16	−13	−22	36	+15	− 4	56	−50	−48
17	+41	+19	37	+ 1	+11	57	−65	−68
18	+25	+33	38	−17	−28	58	− 4	− 3
19	+17	+23	39	−54	−55	59	−50	−35

made. Each value of A is the sum of 6 division error differences, viz. 00′–05′, 10′–15′, 20′–25′, . . . 50′–55′, of which the first and second are tied together, as are also the fourth and fifth.

Referring to the equations on p. 227,

$$(D-d_1)+(d_2-D')=\tfrac{2}{3}(D-D')+\tfrac{1}{3}(x_1-2x_2+x_3)$$
$$(d_1-d_2)=\tfrac{1}{3}(D-D')-\tfrac{1}{3}(x_1-2x_2+x_3).$$

If the probable errors of D, D' are assumed each equal to ϵ_D, and of each of the x's to ϵ_x, it follows, from the way in which A is built up, that

$$\epsilon_A^2=\tfrac{8}{9}\epsilon_D^2+\tfrac{24}{9}\epsilon_x^2$$
$$=\tfrac{8}{9}\epsilon_D^2+4\epsilon_d^2,$$

where ϵ_d is the portion of the probable error of ϵ_d which is independent of the errors of D, D'.

Taking $\epsilon_A=\pm 0''\cdot 052$, $\epsilon_D=\pm 0''\cdot 019$, we obtain

$$\epsilon_d=\pm 0''\cdot 024.$$

The addition of the effect of the uncertainty in the adopted division errors D, D' increases this probable error to $\pm 0''\cdot 028$, which is slightly smaller than but in close agreement with the value previously derived on p. 228.

COMPARISON STARS FOR RS AND NOVA PUPPIS

R. H. Stoy

(Communicated by H.M. Astronomer)

(Received 1944 August 16)

A request from Mr. Cousins for more comparison stars for Nova Puppis led me to remeasure a number of plates that had been taken in connection with the Cape $-30°$ to $-35°$ Zone Catalogue. At the same time it seemed worth while to work up all the information the plates could give about the interesting long-period Cepheid variable RS Puppis which lies close to the nova.

Columns three and four of Table I give the photographic and photovisual magnitudes of 49 stars in the region surrounding RS and Nova Puppis. These magnitudes, which are on the same system as those given in a previous paper *, were derived from measures of five pairs of plates taken with the photometric lenses, the measures being reduced by means of magnitudes taken from the Cape Zone Catalogue. The magnitudes, which are given to one place of decimals only, are extrapolations based on measures of a single pair of plates taken with the Victoria Telescope. The other columns of Table I give the list number, the HD or CD number, the colour index, the HD spectral type and the designations and magnitudes in the list of comparison stars given by Innes †, Gerasimovič ‡, Voûte § and Pettit ||. The final column of Table I refers to a provisional list of comparison stars that was published in the *Monthly Notes of the Astronomical Society of South Africa* in December 1942. These provisional magnitudes, which are roughly on the Harvard Photovisual scale, were derived from measures of three photovisual plates reduced by means of magnitudes given in *Harvard Observatory Mimeograms*, Series III, No. 1, 1938.

No. 2 on Pettit's list is the irregular variable Y Puppis. If the variability of this star be ignored, its measured *IPg*, *IPv* and C.I. are 10·66, 8·92 and 1·74 respectively. Unfortunately the star is near the limit of the photographic plates and is not recorded on those taken in 1940. This is not to be interpreted as meaning that the star was then fainter than in 1939, as the plates used in 1940, though nominally the same, were noticeably less sensitive. Judging from the photovisual plates, the star was actually about $0^m.16$ brighter in 1940 than in 1939.

Table II gives the magnitudes of RS Puppis as measured on twelve pairs of plates taken during 1939 and 1940. By good fortune the plates, which were taken for routine purposes, cover the greater part of the variation of RS Puppis. The phase (maximum at $0^d.00$) according to Voûte and according to Gerasimovič is given in columns two and three. Scrappy though the observations are, they undoubtedly fit Voûte's formula better than they do that of Gerasimovič. The range in light seems to be from $7^m.7$ to $9^m.4$ in the photographic and from $6^m.5$ to $7^m.8$ in the photovisual. The colour index seems to remain fairly constant at about $1^m.6$ except during the nine-day rise to maximum, when it gradually decreases to $1^m.2$.

Voûte's period of 41·414 days is longer than that of 41·337 days found from earlier data by Gerasimovič. The period found by Innes from still earlier data is 41·313 days. The star is being observed photographically at the Lembang and Riverview observatories, but visual observations are also desirable. Suitable comparison stars having a colour index comparable with that of the variable are Nos. 1, 13, 15, 21 and 42 of Table I, provided that the circumstances of observation permit their use.

There are no suitable comparison stars for Nova Puppis now that its light is chiefly that of nebular emissions. Perhaps the best that can be done is to use stars with as small a colour index as possible.

* *M.N.*, **103**, 288, 1943. † *An. Cape Obs.*, **9**, 60B, 1903. ‡ *H.B.*, 848, 1927.
§ *An. Bosscha Ster.*, **8**, 31, 1937. || *P.A.S.P.*, **56**, 32, 1944.

TABLE I

Comparison Stars for RS and Nova Puppis

No.	Star	*IPg*	*IPv*	C.I.	Sp.	Innes	Gerasimovič	Voûte	Pettit	A.S.S.A.
1	69123	6·97	5·75	+1·22	Ko	6·4	a 6·74	a 6·80		*17* 5·67
2	69066	7·79	6·95	+0·84	Ko	7·0	b 7·47	b 7·58	*1* 6·97	*14* 6·81
3	69760	7·65	7·65	0·00	Ao	7·6	c 7·54			
4	69042	8·24	8·15	+0·09	Ao		d 7·95	d 8·25		
5	68734	9·08	9·23	−0·15	Ao		e 8·87	e 9·20		
6	−34° 4503	9·50	9·19	+0·31			f 9·18			
7	−34° 4496	9·51	9·28	+0·23			g 9·95	g 9·56		
8	−34° 4510	9·98	10·11	−0·13			h10·20	h10·11		
9	70530	7·83	8·01	−0·18	B9			c 8·10		
10	68215	8·74	8·58	+0·16	A3			d 8·76		
11	67735	7·96	7·95	+0·01	Ao	7·7				
12	67736	7·60	7·23	+0·37	A5	7·0				*1* 7·03
13	69002	7·77	6·39	+1·38	K2	6·8				
14	70235	6·36	6·50	−0·14	B9	6·7				
15	67977	7·23	6·23	+1·00	G5	6·6				*3* 6·12
16	69280	8·14	6·80	+1·34	K2	7·2				*18* 6·66
17	68363	8·66	7·20	+1·46	K2					
18	66583	9·05	7·31	+1·74	Ma					
19	70869	9·28	7·97	+1·31	K2					
20	−32° 4927	9·56	7·81	+1·75						
21	−33° 4867	9·88	8·24	+1·64						
22	68842	9·09	8·53	+0·56	G5				*3* 8·50	
23	68961	8·73	8·73	0·00	Ao				*4* 8·69	
24	68491	8·84	8·67	+0·17	Fo				*5* 8·79	*9* 8·36
25	68861	9·10	9·00	+0·10	B9				*6* 8·92	
26	−34° 4470	9·59	9·24	+0·35					*7* 9·17	
27	68537	9·10	9·08	+0·02	Ao				*8* 9·19	*10* 8·74
28	68735	9·10	9·21	−0·11	Ao				*9* 9·26	
29	−34° 4446	10·19	9·28	+0·91					*10* 9·29	
30	−35° 4323	10·53	9·57	+0·96					*11* 9·36	
31	−34° 4465	9·59	9·55	+0·04					*12* 9·56	
32	−35° 4294	10·10	9·79	+0·31					*13* 9·74	
33	−34° 4458	9·87	10·08	−0·21					*14* 9·97	
34	−34° 4460	10·68	10·24	+0·44					*15* 10·16	
35	−34° 4450	10·55	10·6	−0·1					*16* 10·36	
36	−35° 4299	12·4	10·4	+2·0					*17* 10·62	
37	68154	8·99	8·19	+0·80	Ko					*4* 7·97
38	68296	8·69	8·85	−0·16	B9					*5* 8·53
39	68297	8·87	8·95	−0·08	Ao					*6* 8·64
40	68341	9·58	9·79	−0·21	A					*7* 9·43
41	68473	9·09	9·22	−0·13	B9					*8* 8·88
42	68623	8·47	6·71	+1·76	Mb					*11* 6·58
43	68676	8·44	7·88	+0·56	G5					*12* 7·63
44	−35° 4305	9·69	9·36	+0·33						
45	−35° 4289	10·7	10·4	+0·3						
46	−35° 4286	11·2	11·0	+0·2						
47	−34° 4439	11·3	10·8	+0·5						
48	−35° 4311	11·6	10·6	+1·0						
49	−35° 4296	12·0	10·8	+1·2						

TABLE II

Magnitudes of RS Puppis

J.D.	Phase		*IPg*	*IPv*	C.I.
	Voûte	Gerasimovič			
2429,000 +					
309·354	30·80	36·83	9·25	7·67	1·58
332·312	12·34	18·45	8·56	6·94	1·62
344·269	24·30	30·40	9·08	7·52	1·56
344·301	24·33	30·44	9·18	7·56	1·62
348·273	28·30	34·41	9·26	7·63	1·63
352·275	32·31	38·41	9·35	7·76	1·59
672·352	21·07	27·79	8·86	7·21	1·65
692·309	41·03	6·41	7·69	6·46	1·23
698·308	5·61	12·41	8·29	6·62	1·67
701·304	8·61	15·41	8·47	6·73	1·74
728·282	35·59	1·05	8·68	7·15	1·53
729·248	36·55	2·01	8·31	6·89	1·42

The Royal Observatory,
Cape of Good Hope:
1944 *May* 16.

OCCULTATIONS OF STARS AND PLANET BY THE MOON OBSERVED AT THE NIZAMIAH OBSERVATORY, HYDERABAD, DURING THE YEAR 1943

(Communicated by the Director)

(Received 1944 June 9)

TABLE I

No.	Star	Mag.	Phase	Date 1943	U.T. h m s
1	29 Piscium	5·2	D	Jan. 11	12 53 06·4
2	BD −3° 5751	9·2	D	,, 11	13 30 11·9
3	BD −3° 5752	8·7	D	,, 11	13 34 27·3
4	BD −3° 5750	6·8	D	,, 11	13 41 34·0
5	4 Ceti	6·3	D	,, 11	16 35 51·7
6	BD +0° 146	9·1	D	,, 12	15 42 30·9
7	BD +0° 149	7·3	D	,, 12	16 12 50·7
8	BD +15° 592	7·2	D	,, 16	13 41 02·0
9	Venus	−3·3	R	Feb. 6	10 57 21·8
10	BD +16° 657	7·2	D	,, 13	17 14 12·6
11	ζ Cancri *m.*	5·1	D	,, 17	16 47 37·4
12	BD +18° 1882	6·4	D	,, 17	18 24 27·9
13	13 Libræ	5·8	R	,, 25	23 54 56·8
14	130 B. Sagittarii	6·5	R	Mar. 1	23 12 31·7
15	BD +18° 1778	7·4	D	,, 16	16 38 25·1
16	γ Libræ	4·0	D	,, 25	23 37 45·1
17	39 G. Sagittarii *m.*	6·3	R	Apr. 24	21 19 58·5
18	BD +12° 2177	7·3	D	June 9	13 45 02·3
19	13 Libræ	5·8	D	July 12	17 06 01·3
20	21 Sagittarii	5·0	D	Aug. 12	16 32 57·1
21	*f* Sagittarii	5·1	D	Oct. 7	16 34 34·2
22	45 Capricorni	5·9	D	,, 9	15 20 52·7
23	33 Ceti	6·2	D	Nov. 9	17 02 05·0
24	85 Ceti	6·3	D	Dec. 8	19 57 07·0
25	ζ Gemini	4·0	D	,, 13	15 16 43·5
26	ζ Gemini	4·0	R	,, 13	16 16 39·3
27	217 B. Gemini	6·3	R	,, 14	19 04 04·6
28	BD −10° 6008	6·8	D	,, 31	15 55 39·0

All the observations were made by M. K. Bappu. The instrument used throughout was the 15-inch visual refractor. The corrections to the standard clock were determined from observations with the transit instrument.

The reductions were carried out by M. V. V. Sastry assisted by S. Aravamudan. No correction was added to the observed time.

TABLE II

No.	$\sigma' - \sigma$	$\cos(\rho - \chi)$	$\sin(\rho - \chi)$	No.	$\sigma' - \sigma$	$\cos(\rho - \chi)$	$\sin(\rho - \chi)$
	″				″		
1	+0·9	+0·95	+0·32	15	+1·0	+0·82	−0·58
2	[+4·6]	+0·63	+0·78	16	+0·7	+0·60	−0·80
3	+2·9	+0·98	+0·22	17	−1·4	−0·31	+0·95
4	+0·9	+0·34	+0·94	18	+2·6	+0·76	−0·65
5	+2·8	+0·78	−0·63	19	+1·0	+0·91	−0·41
6	+2·4	+0·67	−0·74	20	−2·3	+0·96	−0·27
7	+2·2	+0·71	−0·70	21	+0·8	+0·93	+0·37
8	−0·2	+0·38	+0·92	22	−0·8	+0·65	+0·76
9	−3·3	−0·65	−0·76	23	−0·1	+0·91	+0·41
10	+1·3	+0·59	−0·81	24	−0·3	+0·25	+0·97
11	−1·3	+0·93	−0·37	25	−0·6	+0·77	+0·63
12	+0·2	+0·97	+0·23	26	0·0	−0·94	+0·35
13	−0·7	−0·82	−0·57	27	+0·8	−0·83	+0·56
14	−0·2	−0·85	+0·53	28	−0·3	+0·68	+0·73

The mean places for 1943·0 of three stars in the list are given in the following table. For stars Nos. 2 and 6 proper motions have been applied to the co-ordinates given in the catalogue in computing the mean places for use in the reductions. For the remaining stars, positions were taken from the *Nautical Almanac*.

TABLE III

No.	R.A. (1943·0)	Dec. (1943·0)	Remarks
	h m s	° ′ ″	
2	23 59 13·03	−3 08 06·2	San Fernando −3°, $23^h\ 56^m$ 104; −4°, $0^h\ 0^m$ 17 ; P.M. μ_α −0ˢ·0013, μ_δ −0″·016.
3	23 59 51·89	−3 13 59·6	A.G. Strassburg 8198.
6	0 53 55·32	+1 22 22·9	Algiers +1°, $0^h\ 52^m$ 28; P.M. μ_α +0ˢ·0010, μ_δ 0″.

Nizamiah Observatory,
Hyderabad, Deccan, India:
1944 *April.*

ERRATA

M.N., **104**, No. 3:

S. Vsessviatsky and E. Bougoslavsky, *The Solar Eclipse of* 1936 *June* 19.

The Editors regret that several small corrections were received from the authors too late for incorporation in this paper.

For Bougoslavsky *read* Bougoslavskaya (throughout);
P. 141, line 4, *for* Azambuja *read* d'Azambuja;
P. 143, 18 lines from bottom, *delete* (see Fig. 3);
17 lines from bottom, after the word prominence, *insert* (see Plates 1 and 2);
P. 144, line 9, *for* (see Plates) *read* (see Fig. 3).

M.N., **104**, No. 3, G. Alter's paper:

P. 186, Table VI. The Editors regret that two blocks of figures have been transposed in this table; the stars should be arranged in order of increasing $m - M$, *i.e.* the block of figures for the stars 24–149 in the left-hand side of the table should be interchanged with the block for the stars 42–87 in the right-hand side of the table.